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Planning Fuel-Conservative Descents in an Airline Environment Using a Small Programmable Calculator

Algorithm Development and Flight Test Results

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Algorithm Development and Flight Test Results

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## Summary

The Federal Aviation Administration (FAA) is implementing an automated, time-based metering form of air traffic control with profile descent procedures for arrivals into the terminal area. These concepts provide fuel savings by matching the arrival flow of airplanes to the airport acceptance rate through time-control computations and by allowing the pilot to descend at his discretion from cruise altitude to a designated metering fix in an idle-thrust clean configuration (with landing gear up, flaps zero, and speed brakes retracted). Although substantial fuel savings have resulted from these procedures, a potential for further fuel savings exists. Currently, the radar controller maintains time management for each airplane through either speed control or path stretching with radar vectors. This often results in more fuel being consumed than would have been if the descent planned by the pilot had been flown. Work load for the pilot is also high since he must plan for an idle-thrust descent to the metering fix by using various rules of thumb.

The National Aeronautics and Space Administration (NASA) has developed an airborne descent algorithm compatible with time-based metering and profile descent procedures and designed to improve the accuracy of delivering an airplane during a fuelefficient descent to a metering fix at a time designated by the air traffic control (ATC) system. This algorithm provides open-loop guidance for an airplane to make an idle-thrust, clean-configured descent to arrive at the metering fix at a predetermined time, altitude, and airspeed. The algorithm also provides open-loop guidance for fuel-conservative descents when time constraints are not a consideration.

An investigation into the feasibility of using the open-loop guidance in an airline operational environment was conducted. The algorithm was programmed on a small programmable calculator for use with a McDonnell Douglas DC-10-10 airplane. Flight tests were conducted on routine airline flights into various major airports. The resulting mean distance and time errors to actually achieve the predicted speed and altitude conditions at the end of the descent profile were $2.3 \mathrm{n} . \mathrm{mi}$. long and 1.3 sec early, respectively, based on 19 test runs. The mean arrival time error at the metering fix was 8.4 sec early (compared with approximately 1 to 2 min when vectored by ATC). The subject pilots reported that the calculator was easy to use and did not interfere with normal flight duties. They felt that the open-loop guidance provided by the calculator would be of most value while flying descents at nonstandard speeds or during time-metered operations.

## Introduction

In an effort to improve the efficiency of terminal area operations, the Federal Aviation Administration (FAA) is implementing an automated time-based metering form of air traffic control (ATC) with profile descent procedures. The time-based metering concept is based upon airplane arrivals crossing a metering fix (typically 30 to $40 \mathrm{n} . \mathrm{mi}$. from the airport) at a specified altitude, airspeed, and time. The time metering derandomizes the arrivals to the airport prior to entering the terminal area and results in a reduction of the low-altitude, high-fuel-consumption flight normally used to sequence airplanes to a common final approach path. The proper sequencing and spacing of enroute traffic also allows for increased airport productivity (refs. 1 and 2). The profile descent procedure allows the pilot to plan and fly a descent that is fuel conservative for his particular airplane, thus resulting in additional fuel savings.

With the current time-based metering/profile descent procedures, the air traffic controller is responsible for the time management of each airplane. The controller can adjust the airplane time of arrival at the metering fix by increasing the flight path length through heading changes or by requesting the pilot to change speed. The pilot is responsible for crossing the metering fix at the proper airspeed and altitude and must plan the descent carefully if fuel is to be conserved. The time management and the descent planning are both work load intensive and interdependent. Since limited or no guidance is available to either the controller or the pilot, their tasks must be accomplished independently through various rules of thumb and past experience. With this present operational concept, airplanes typically cross the metering fix with a time accuracy between 1 and 2 min (ref. 3). When the controller must lengthen the path or change the speed of the airplane for time control (even though the time-based metering and profile descent procedures are saving fuel), additional fuel must be used.

During the summer of 1979, the National Aeronautics and Space Administration (NASA) developed and flight tested, in its Transport Systems Research Vehicle (TSRV, previously designated the Terminal Configured Vehicle) Boeing 737 research airplane, a flight-management descent algorithm designed to provide closed-loop guidance for a fuelconservative descent and to reduce the metering-fix crossing-time dispersion. The flight-management algorithm, which was based on an idle-thrust, cleanconfigured descent (with landing gear up and flaps and spoilers retracted), was designed to place the airplane at the metering fix at the proper altitude,
airspeed, and ATC-designated time. The results of the flight tests showed that the closed-loop guidance provided by the guidance and display system could reduce the time error when the metering fix is crossed to approximately 12 sec , based on 19 test runs (ref. 4).

This research was continued in June of 1981 with a T-39A (Sabreliner) airplane to determine if similar results could be obtained with open-loop guidance and a conventional complement of cockpit instrumentation (ref. 5). A version of the flight-management descent algorithm was implemented on a HewlettPackard HP-41CV programmable calculator. Openloop guidance was provided by the calculator in the form of the Mach number and airspeed at which the descent should be flown and the point at which the pilot was to reduce the thrust to flight idle and begin the descent. The descent was then flown with reference to the airplane Mach and airspeed indicators and by maintaining an altitude profile computed by the calculator as a function of distance to the metering fix. Flight tests using this open-loop guidance resulted in an average time error when crossing the metering fix of 20 sec , based on 12 runs. This version of the descent algorithm was designed to operate in both the time-based metered and the conventional air traffic control environment, in which metering-fix times are not assigned.

Having determined the viability of the open-loop descent guidance, additional research was conducted to evaluate the feasibility of using such a descent planning tool in an airline operational environment. The airplane used in this study was the McDonnell Douglas DC-10-10. This report contains a description of the programmable calculator software, the DC-10 descent performance model used in the algorithm computations, and the results of the model-validation and pilot-evaluation flight tests.

## Symbols and Abbreviations

| ATC | air traffic control |
| :--- | :--- |
| $A_{\mathrm{gw}}$ | coefficient for gross-weight multiplica- <br> tion factor, $\mathrm{lb}^{-1}$ |
| $A_{I}$ | coefficient for constant IAS descent <br> rate equation, $\mathrm{ft} / \mathrm{sec}$ |
| $A_{M}, B_{M}$ | coefficients for constant Mach descent <br> rate equation, ft |
| $a_{0}, a_{1}, a_{2}$ | coefficients for quadratic curve fits of <br> altitude as a function of time |
| $B_{\mathrm{gw}}$ | coefficient for gross-weight multiplica- <br> tion factor |


| $B_{I}$ | coefficient for constant IAS descent rate equation, knots ${ }^{-1}$ |
| :---: | :---: |
| $b_{0}$ | modeled vertical speed at sea level, $\mathrm{ft} / \mathrm{sec}$ |
| $b_{1}$ | slope of linear model for $\dot{h}$ during constant IAS descent, ( $\mathrm{ft} / \mathrm{sec}$ )/ft |
| CRS | magnetic course from entry fix to metering fix, deg |
| $C_{I}$ | coefficient for constant IAS descent rate equation, $\mathrm{sec}^{-1}$ |
| $C_{M}$ | coefficient for constant Mach descent rate equation, $\mathrm{sec}^{2} / \mathrm{ft}$ |
| $c_{0}$ | constant in model for $\dot{h}_{M_{d}}, \sec ^{2} / \mathrm{ft}$ |
| $c_{1}$ | constant in model for $\dot{h}_{M_{d}}, \mathrm{ft}$ |
| DME | distance measuring equipment |
| $D_{w}$ | magnetic wind direction, deg |
| $D_{w, h}$ | magnetic wind direction evaluated at altitude $h$, deg |
| $D_{w, s}$ | magnetic wind direction computed for sea-level altitude, deg |
| $d D_{w} / d H$ | wind direction gradient with respect to altitude $H, \mathrm{deg} / \mathrm{ft}$ |
| EF | entry fix |
| EF ${ }_{\text {DME }}$ | DME reading at entry fix, n.mi. |
| GSc | ground speed at cruise altitude, knots |
| $\mathrm{GS}_{5}$ | average ground speed on segment 5 , knots |
| GW | gross weight, lb |
| $H$ | pressure altitude, ft |
| $H_{\mathrm{av}}$ | average pressure altitude, ft |
| $H_{\text {bod }}$ | pressure altitude at bottom of descent, ft |
| $H_{c}$ | pressure altitude at cruise, ft |
| $H_{\text {MF }}$ | pressure altitude of metering fix, ft |
| $H_{\mathrm{XO}}$ | pressure altitude at transition from constant Mach descent to constant airspeed descent, ft |
| $h$ | geopotential altitude, ft |
| $h_{c}$ | cruise altitude, ft |
| $h_{\text {MF }}$ | metering-fix altitude, ft |



| $\mathrm{T}_{\text {ISA }, H}$ | International Standard Atmospheric temperature at altitude $H, \mathrm{~K}$ |
| :---: | :---: |
| $T_{o}$ | standard sea-level air temperature, K |
| $T_{o}^{\prime}$ | nonstandard sea-level air temperature, K |
| $T_{\text {st }, c}$ | static air temperature measured at cruise altitude, K |
| $T_{\text {st, },}$ | static air temperature at altitude $H$, K |
| $T_{\text {trop }}$ | static air temperature at tropopause, K |
| $\Delta T$ | difference between actual temperature and standard temperature, K |
| $t$ | time, sec |
| $t_{E}$ | time error for descent speed convergence criterion, sec |
| $t_{E, \text { initial }}$ | initial time error, sec |
| $t_{\text {EF }}$ | time that entry fix was crossed, hr:min:sec |
| $t_{\text {IDL }}$ | crossing time of point at which throttles are reduced to flight idle, hr:min:sec |
| $t_{\text {MF }}$ | metering-fix crossing time, hr :min:sec |
| $\Delta t$ | time increment, sec |
| $\Delta t_{\text {initial }}$ | time required to fly initial descent profile, sec |
| $\Delta t_{j}$ | time required to fly on path segment $j$, sec |
| $\Delta t_{\text {req }}$ | time required to fly between entry fix and metering fix, sec |
| VAR | magnetic variation in descent area, deg |
| $V$ | indicated airspeed used in flow chart |
| $V_{f}$ | final speed of level flight segment, knots |
| $V_{i}$ | initial speed of level flight segment, knots |
| $W_{c}$ | difference between actual and computed ground speeds at cruise altitude, knots |
| $W_{H, h}$ | head-wind component along airplane ground track evaluated at altitude $h$, knots |


| $W_{H, h_{c}}$ | head-wind component along airplane <br> ground track in cruise, knots |
| :--- | :--- |
| $W_{H, h_{M F}}$ | head-wind component along airplane <br> ground track at metering fix, knots |
| $X$ | distance variable in DME $\rightarrow H$ routine, <br> n.mi. |
| $X_{\text {DME }}$ | input DME reading used in DME $\rightarrow H$ <br> routine, n.mi. |
| $\ddot{x}$ | acceleration, knots/sec |
| $Y_{1}, Y_{2}, Y_{3}$ | substitution variables used in <br> DME $\rightarrow H$ routine |

Airport abbreviations:

| BDL | Bradley International, Windsor Locks, <br> Connecticut |
| :---: | :--- |
| BOS | Logan International, Boston, <br> Massachusetts |
| DEN | Stapleton International, Denver, <br> Colorado |
| EWR | Newark International, Newark, New <br> Jersey |
| HNL | Honolulu International, Honolulu, <br> Hawaii |
| LAX | Los Angeles International, Los Ange- <br> les, California |
| ORD | O'Hare International, Chicago, Illinois <br> San Francisco International, San |
| SFO | Francisco, California |

## Description of Flight-Management Descent Algorithm

## Description of General Profile

The flight-management descent algorithm computes the parameters required to describe a sevensegment cruise and descent profile (fig. 1) between an arbitrarily located entry fix and an ATC-defined metering fix. The descent profile is computed based on empirical modeling of airplane performance for an idle-thrust, clean-configured descent. The descent Mach/airspeed schedule, airplane gross weight, wind, wind gradient, and nonstandard temperature effects are also considered in these calculations.

Figure 1 shows the vertical-plane geometry of the path between the entry fix and the metering fix. Each path segment, starting at the metering fix, is numbered according to the order in which it
is calculated by the algorithm. To be compatible with standard airline operating practices, the path is calculated based upon the descent being flown first at a constant Mach number with a subsequent transition to a constant indicated airspeed and with all speed reductions made in level flight.

The first segment traversed on the profile is segment 7, which begins at the entry fix and is flown at constant cruise altitude and Mach number. Segment 6 is a relatively short, level-flight path segment in which the pilot reduces thrust to flight idle so that the airplane will slow from the cruise Mach number to the descent Mach number. Segment 6 is eliminated if the descent and cruise Mach numbers are the same. Once the descent Mach number is attained, the constant Mach descent segment (segment 5) is started. As altitude is decreased along this path segment, the indicated airspeed will increase because of increasing air pressure. Segment 4 begins when the desired indicated airspeed is attained for descent. The descent is continued along this segment at the desired, constant indicated airspeed. When the metering-fix altitude has been reached, the airplane is flown at a constant altitude along segment 3 and is slowed from the descent airspeed to the designated airspeed over the metering fix. Segments 1 and 2 are not computed.

If the metering fix is below 10000 ft MSL and the descent airspeed flown on segment 4 is greater than 250 knots, the airplane must be slowed to comply with an ATC-imposed speed limit of 250 knots below 10000 ft MSL. In this case, segments 1 and 2 are computed as depicted in figure 1. Segment 3 then becomes a level-flight segment at 10000 ft MSL in which the airspeed is reduced to 250 knots. The descent is then continued at 250 knots along segment 2. When the metering-fix altitude has been reached, the airplane is flown at a constant altitude along segment 1 and is slowed from 250 knots to the designated airspeed over the metering fix. Path segment 1 is not computed if the metering-fix crossing airspeed is also 250 knots.

The flight-management descent algorithm can be used in either of two modes. The first mode was designed for time-metered operations. In this mode, instead of specifying the $M$ /IAS descent schedule, the pilot enters the time that the entry fix was crossed and the metering-fix arrival time that was assigned by ATC. The descent profile is then calculated based on an $M$ /IAS descent schedule, which is computed through an iterative process, that will closely satisfy the crossing time specified for the metering fix. During this iterative process, the Mach number used for the descent is set equal to the cruise Mach number in an effort to reduce computational requirements and operational complexity. The airspeed is adjusted to
satisfy the time constraints. A check is made within the profile computations to ensure that the descent airspeed $\mathrm{IAS}_{\boldsymbol{d}}$ is within the minimum and maximum speed limits for the particular airplane modeled. For the DC-10 airplane, these limits were

$$
220 \leq \mathrm{IAS}_{d} \leq 350 \quad[\mathrm{knots}]
$$

An additional constraint was that $\mathrm{IAS}_{d}$ would not be less than the airspeed at which the airplane was to cross the metering fix. This eliminated the need to accelerate the airplane to a higher airspeed, which results in greater fuel usage. If the ATC-assigned metering-fix crossing time requires a descent airspeed less than the airplane minimum descent airspeed limit, the profile is computed based on the minimum airspeed limit and a message is displayed to the pilot to "hold" (delay) for the required amount of time. A similar "late" message is displayed with the time error if a descent airspeed schedule greater than the maximum allowed is required.

The second mode is called the speed mode. In this mode the pilot must enter the desired M/IAS to be flown during descent. The descent profile is then computed, based on this descent speed schedule, without consideration of a constraint on metering-fix arrival time. This mode would be used when timebased metering is not being used.

## Logic Flow of Profile Descent Algorithm

Figure 2 shows the general logic flow of the profile descent computations. Pilot inputs used to compute the profile may be entered prior to flight and modified, as required, prior to the descent. These parameters include cruise altitude and Mach number, airplane gross weight, outside air temperature, wind velocity at various altitudes selected by the pilot, entry-fix and metering-fix descriptions, and the course direction to the metering fix. In addition to these parameters, the pilot may enter either a particular Mach number and indicated airspeed to be used during the descent or the entry-fix crossing time and the ATC-assigned metering-fix crossing time.

If the $M /$ IAS descent speed schedule has been entered in the calculator, the computations will be based on a nonmetered traffic environment. The pilot initiates the computations by pushing the "compute" key. The descent profile is then computed in a single iteration, and the point where thrust should be reduced to flight idle to start the descent is indicated as a DME distance on the calculator display.

If the entry-fix crossing time and the ATCassigned metering-fix crossing time have been entered in the calculator, the time required to fly between the fixes $\Delta t_{\text {req }}$ will be computed and subsequent calcu-
lations will be based on a time-metered traffic environment. Once the pilot has initiated the computations by pushing the "compute" key, an iterative process is started to determine an appropriate IAS descent speed that will satisfy the time constraints. The Mach number used for the descent is set equal to the cruise Mach number in an effort to reduce computational requirements and operational complexity.

The iterative process starts with the computation of the time to fly from the entry fix to the metering fix $\left(\sum_{j=1}^{7} \Delta t_{j}\right)_{\text {initial }}$ at the following descent speed schedule:

$$
\begin{aligned}
& M_{d, \text { initial }}=M_{c} \\
& \mathrm{IAS}_{d, \text { initial }}=\left\{\begin{array} { l l } 
{ 2 8 0 } & { [ \text { knots } ] }
\end{array} \left(\begin{array} { l l } 
{ \text { (if } \mathrm { IAS } _ { \mathrm { MF } } \leq 2 8 0 ) } \\
{ \mathrm { IAS } _ { \mathrm { MF } } } & { [ \text { knots } ] }
\end{array} \left(\begin{array}{ll}
\text { (if } \left.\mathrm{IAS}_{\mathrm{MF}}>280\right)
\end{array}\right.\right.\right.
\end{aligned}
$$

The IAS $_{d, \text { initial }}$ value of 280 knots is used because it is the approximate midpoint between the maximum and minimum allowable descent airspeeds and because it is a descent speed typically used by the airlines.

A check is made with the following transitiontime inequality to determine if the time-convergence criterion $t_{E}$ has been satisfied. (For the purposes of these tests, $t_{E}=5 \mathrm{sec}$.)

$$
\left|\Delta t_{\mathrm{req}}-\sum_{j=1}^{7} \Delta t_{j}\right| \leq t_{E} \quad[\mathrm{sec}]
$$

where

$$
\Delta t_{\mathrm{req}}=t_{\mathrm{MF}}-t_{\mathrm{EF}} \quad[\mathrm{sec}]
$$

If this inequality is satisfied, the computations are complete and the idle-thrust descent point is displayed to the pilot. If the inequality is not satisfied, the descent computations will be repeated by using the operational airspeed limits as follows:

$$
\begin{gathered}
M_{d}=M_{c} \\
\mathrm{IAS}_{d}=\left\{\begin{array}{lll}
350 & {[\mathrm{knots}]} & \left(\text { if } \Delta t_{\mathrm{req}}<\left(\sum_{j=1}^{7} \Delta t_{j}\right)_{\text {initial }}\right) \\
\mathrm{IAS} & {[\mathrm{knots}]} & \text { otherwise }
\end{array}\right.
\end{gathered}
$$

A check is then made to determine if the time criterion has been satisfied or if a speed greater than 350 knots or less than IAS $_{\text {MF }}$ would be required to satisfy the time constraints. If the time criterion is satisfied, the idle-thrust descent point is displayed to the pilot. If either the upper or lower airspeed limit must be violated to satisfy the time constraints, the appropriate speed limit will be used in the descent
computations and the resulting time error for crossing the metering fix will be displayed to the pilot.

If the time criterion has not been satisfied and neither the upper nor lower airspeed limitation will be violated, a revised descent airspeed IAS $_{d}$ and associated descent time will be computed and compared with $\Delta t_{\text {req }}$. This iterative process will continue until the time-convergence criterion has been satisfied.

The computation of the revised IAS $_{d}$ is graphically depicted in figure 3, which shows a plot of the time required to fly between a specified entry fix and metering fix at a specified cruise Mach number over the complete $\mathrm{IAS}_{d}$ range of the airplane. The descent airspeed is revised through a modified linear interpolation of the desired $\Delta t_{\text {req }}$ within a range of time bounded by an initial value and a computed variable value. The initial value $\left(\sum_{j=1}^{7} \Delta t_{j}\right)_{\text {initial }}$ was the resulting time computed with the $M /$ IAS $_{d, \text { initial }}$ descent speed schedule on the first iteration. The variable time value $\left(\sum_{j=1}^{7} \Delta t_{j}\right)_{i-1}$ is the time computed for the $M /$ IAS $_{d, i-1}$ descent speed schedule on the last ( $i-1$ ) iteration. The revised descent speed schedule $M / \mathrm{IAS}_{d, i}$ is computed as follows:

$$
M_{d}=M_{c}
$$

$$
\begin{aligned}
\mathrm{IAS}_{d, i}= & \mathrm{IAS}_{d, \text { initial }}+\left(\mathrm{IAS}_{d, i-1}-\mathrm{IAS}_{d, \text { initial }}\right) K \\
& -\frac{\mathrm{IAS}_{d, i-1}-\mathrm{IAS}_{d, \text { initial }}}{\left(\sum_{j=1}^{7} \Delta t_{j}\right)_{\text {initial }}-\left(\sum_{j=1}^{7} \Delta t_{j}\right)_{i-1}} \\
& \times 5 \sin (180 K) \quad[\mathrm{knots}]
\end{aligned}
$$

where

$$
K=\frac{\left(\sum_{j=1}^{7} \Delta t_{j}\right)_{\mathrm{initial}}-\Delta t_{\mathrm{req}}}{\left(\sum_{j=1}^{7} \Delta t_{j}\right)_{\mathrm{initial}}-\left(\sum_{j=1}^{7} \Delta t_{j}\right)_{i-1}}
$$

and $i$ is the $i$ th iteration.
The last term of the computations for IAS $_{d, i}$ is a compensation factor for the difference in curvature between the plot of time required to fly between the entry fix and the metering fix as a function of descent airspeed and the straight line used in the linear interpolation.

The following numerical example will illustrate the interpolation process. The entry fix is crossed at an altitude of 37000 ft and at a cruise Mach number of 0.81 . ATC has assigned a metering-fix crossing time for $15 \mathrm{~min}(900 \mathrm{sec})$ later at an altitude of 8000 ft and at an airspeed of 280 knots.

An initial airspeed of 280 knots has been programmed into the interpolation routine. At this
speed, 934.8 sec are needed to fly this profile. Since the ATC time required to fly the profile is 900 sec , the maximum limit airspeed of 350 knots is chosen by the calculator for the second iteration. The time needed to fly the profile with a descent airspeed of 350 knots was computed to be 842.5 sec .

An interpolation for the next descent airspeed can now be made between the speed and time points of 280 knots and 934.8 sec and of 350 knots and 842.5 sec , respectively. A simple linear interpolation between these points for 900 sec results in a speed of 306.4 knots. However, when the compensation factor term is applied to correct for the straightline interpolation effects, a slightly lower airspeed of 302.9 knots is obtained for further computations.

At a descent airspeed of 302.9 knots, 890.2 sec is needed to fly the profile. Since this exceeds the 5 -sec time-convergence criterion ( $900-890.2=9.8$ sec ), another iteration is necessary to select a new descent airspeed. A linear interpolation between the speed and time points of 280 knots and 934.8 sec and of 302.9 knots and 890.2 sec , respectively, results in an airspeed of 297.9 knots for a time of 900 sec . A slightly lower airspeed of 296.2 knots is obtained after the compensation factor term is applied. The time needed to fly the profile with a descent airspeed of 296.2 knots is 901.4 sec . This satisfies the 5 -sec time criterion ( $900-901.4=-1.4 \mathrm{sec}$ ) and completes the iteration process. The descent airspeed displayed to the pilot is 296 knots.

## Empirical Representation of Airplane Performance Characteristics

Computer memory limitations with the programmable calculator preclude the use of detailed aerodynamic and performance tables to represent the airplane for profile descent calculations. Instead, an empirical model of the performance of the DC-10 airplane was developed from flight data collected during idle-thrust, clean-configured descents and during level-flight speed reductions.

A portable voice recorder, a stop watch, and conventional flight instruments were used to collect data for descent performance modeling. Altitude, speed, temperature, and time were recorded at altitude increments of approximately 500 ft during the descents. Speed, temperature, and time were recorded at $10-\mathrm{sec}$ intervals during constant-altitude speed changes. Gross weights were recorded at the beginning and end of each test run.

Constant IAS descent model. The performance model for the constant indicated airspeed descents consisted of linear approximations of vertical speed as
a function of an approximated geopotential altitude for a range of airspeeds between 220 and 350 knots. The vertical speed was adjusted to compensate for variations in gross weight.

The first step in the development of the vertical performance model for constant $\mathrm{IAS}_{d}$ was to approximate, for each descent, geopotential altitude as a function of time as a quadratic equation through a least-squares curve-fit analysis. The general form assumed for this equation was

$$
h=a_{2} t^{2}+a_{1} t+a_{0} \quad[\mathrm{ft}]
$$

Figure 4 shows a typical plot of the data and resulting curve fit for a descent flown at a constant $\mathrm{IAS}_{d}$ of 280 knots.

Vertical speed was determined by differentiating this equation with respect to time. This resulted in an equation of the form

$$
\dot{h}=2 a_{2} t+a_{1} \quad[\mathrm{ft} / \mathrm{sec}]
$$

A plot of $\dot{h}$ as a function of $h$ was then developed for each descent. Figure 5 shows typical data for various descent speeds. These plots indicated that vertical speed was approximately linear as a function of geopotential altitude and were modeled with an equation of the form

$$
\cdots \dot{h}=b_{1} h+b_{0} \quad[\mathrm{ft} / \mathrm{sec}]
$$

The slope $b_{1}$ was approximately the same for all descent speeds and was equal to $-3.5 \times 10^{-4}$ $(\mathrm{ft} / \mathrm{sec}) / \mathrm{ft}$. The modeled vertical speed at sea level $b_{0}$ was corrected for gross-weight variations by dividing by the gross-weight multiplication factor $K_{\mathrm{gw}}$ (to be discussed subsequently). The $b_{0}$ data varied exponentially as a function of descent airspeed. A plot of these data with the exponential model of $b_{0}$ is shown in figure 6.

The vertical-speed model was also adjusted with a correction factor $\dot{h}_{g}$ due to head-wind gradient (to be discussed subsequently) to account for changes in the vertical speed due to head-wind component variations during the descent. The resulting model of vertical speed, for a DC-10 airplane, corrected for gross-weight variations and head-wind gradient effects, was

$$
\begin{aligned}
\dot{h}_{\mathrm{IS}_{d}}= & \left(-3.5 \times 10^{-4}\right) h \\
& +K_{\mathrm{gw}}\left[\dot{h}_{g}-3.07783 \exp (8.158681\right. \\
& \left.\left.\times 10^{-3} \times \mathrm{IAS}_{d}\right)\right] \quad[\mathrm{ft} / \mathrm{sec}]
\end{aligned}
$$

Constant Mach number descent model. The performance model for constant Mach number descents consisted of parabolic approximations of vertical speed as a function of approximated geopotential altitude for a range of Mach numbers between 0.73 and 0.85 . The vertical speed was adjusted to compensate for variations in gross weight.

The procedures used to develop the descent model for constant Mach number were similar to those used for development of the model for constant IAS descents. A quadratic equation for altitude as a function of time was derived for each descent through a least-squares curve-fit analysis. These equations were differentiated with respect to time to determine vertical speed as a function of time. The vertical speed was adjusted for gross-weight variations by dividing by the gross-weight correction factor $K_{\mathrm{gw}}$ (to be discussed subsequently).

The vertical speed $\dot{h}$, adjusted for gross-weight variation, was then plotted as a function of altitude. Typical plots of these data are shown in figure 7. The resulting curves were parabolic in nature and were modeled with an equation of the following form:

$$
\dot{h}_{M_{d}}^{\prime}=-\left[\left(h-c_{1}\right) / c_{0}\right]^{1 / 2} \quad[\mathrm{ft} / \mathrm{sec}]
$$

The magnitude of the coefficient $c_{0}$ was subjectively selected based on the shape of a generic parabola that would overlay the descent data. A value of $c_{0}=-1.85 \mathrm{sec}^{2} / \mathrm{ft}$ was selected and resulted in the parabola shown by the dashed line in figure 7.

The coefficient $c_{1}$ was calculated for each descent at an altitude approximately 2000 ft below cruise altitude for $c_{0}=-1.85 \mathrm{sec}^{2} / \mathrm{ft}$. The resulting values for $c_{1}$ are plotted as a function of the descent Mach number in figure 8. A linear regression analysis resulted in an equation of $c_{1}$ as a function of the Mach number. However, the slope of this model was increased slightly to ensure that $c_{1}$ would be greater than the maximum cruise altitude of the airplane (defined by the altitude limits shown in fig. 8), thus ensuring that an imaginary root would not be obtained from the equation for $\dot{h}_{M_{d}}$. The maximum altitude limits are defined by the maximum operating altitude ( 42000 ft ) of the airplane and by the altitudes corresponding to a minimum airspeed of 220 knots for Mach numbers less than 0.79. The resulting model for $c_{1}$ is

$$
\begin{equation*}
c_{1}=25750 M_{d}+22167 \tag{ft}
\end{equation*}
$$

The correction factor $\dot{h}_{g}$ due to head-wind gradient was added to the descent-rate equation similarly as in the constant indicated airspeed case. The
resulting equation for vertical speed for a constant Mach number descent was

$$
\dot{h}_{M_{d}}=-K_{\mathrm{gw}}\left[\frac{h-\left(25750 M_{d}+22167\right)}{-1.85}\right]^{1 / 2}-\dot{h}_{g} \quad[\mathrm{ft} / \mathrm{sec}]
$$

Acceleration performance model. Acceleration performance data were obtained for idle-thrust, clean-configured speed reductions on level flight paths for typical cruise and metering-fix altitudes. Indicated airspeed and time data were recorded during the speed reductions. The indicated airspeeds were converted to approximate true airspeeds to reduce computational requirements within the descent algorithm. Figure 9 shows a plot of approximate true airspeed, as a function of time, that resulted during speed reductions at various altitudes and gross weights. The average slope of each of these test runs was approximately the same. Hence, acceleration for the DC-10 airplane was modeled as constant and was approximated by

$$
\ddot{x}=-1.3 \quad[\mathrm{knots} / \mathrm{sec}]
$$

Gross-weight variation. The effects of grossweight variation on the descent performance of the airplane were accounted for with a single multiplication factor applied to the vertical performance models developed for both constant airspeed and constant Mach number descents. The multiplication factor $K_{\mathrm{gw}}$ is a linear, nondimensional expression.

The multiplication factor was derived for the DC-10 airplane by plotting the vertical speeds obtained during descents (conducted at the same indicated airspeed) against the gross weight of the airplane. Figure 10 shows the vertical speed at sea level as a function of gross weight for descents conducted at constant airspeeds of $250,280,300$, and 340 knots. A linear curve fit was applied to the data points for the 280 -knot descents since more descents were flown at this speed and since a wider range of gross weights existed in the data. This plot also shows that the model derived for the 280 -knot descents may be shifted vertically (maintaining approximately the same slope) to overlay the descent data obtained at the other speeds. Since the same slope could be approximated for all airspeeds, changes to the vertical speed due to gross-weight variations were modeled independent of airspeed. The plot in figure 10 was then nondimensionalized by dividing the abscissa by 304000 lb and the ordinate by $-30.2 \mathrm{ft} / \mathrm{sec}$ (in which $b_{0}=-30.2 \mathrm{ft} / \mathrm{sec}$ at an airspeed of 280 knots and a gross weight of 304000 lb ). Nondimensionalization allowed the gross-weight variation model derived with the 280 -knot descent data to be expressed in a
form useful for descents at any airspeed. The resulting multiplication factor $K_{\mathrm{gw}}$ for gross-weight variations was

$$
K_{\mathrm{gw}}=\left(-3.863133 \times 10^{-6}\right) \mathrm{GW}+2.174392369
$$

where

$$
260000 \leq \mathrm{GW} \leq 350000 \quad[\mathrm{lb}]
$$

Head-wind gradient effect. The head-windgradient effect is an adjustment to the vertical-speed model required when the head-wind component of the airplane changes during an idle thrust, constant airspeed, or Mach number descent. If the head-wind component decreases during the descent, indicated airspeed will also decrease unless the pilot lowers the pitch angle of the airplane to maintain the airspeed. When the pitch angle is lowered, vertical speed will increase. Similarly, vertical speed must be decreased if the head-wind component increases during the descent.

The head-wind-gradient effect was quantified with an adjustment factor $K_{g}$ computed from data obtained during a series of piloted descents with a DC-10 simulator. The idle-thrust descents were conducted at a constant indicated airspeed of 280 knots, at a gross weight of 300000 lb , and in the presence of head-wind components that decreased 2, 3, and 10 knots per 1000 ft of descent. A quadratic regression analysis was applied to the data from each descent to obtain an equation for altitude as a function of time. The derivative of this equation resulted in a computed, smoothed vertical speed for each of the descents. The average vertical speed obtained for each of the descents with the wind gradients was then subtracted from the average vertical speed obtained during a descent with no wind gradient. The difference in vertical speed was divided by the magnitude of the wind gradient encountered during the descent. The results of these computations were then averaged to obtain a head-wind-gradient factor equal to a change in vertical speed of $-1.28 \mathrm{ft} / \mathrm{sec}$ for a $1-\mathrm{knot}$ decrease in head-wind component for each 1000 ft of change in altitude (i.e., $K_{g}=-1280(\mathrm{ft} / \mathrm{sec}) / \mathrm{knots} / \mathrm{ft}$ ). The effect that the head-wind gradient had on vertical speed $\dot{h}_{g}$ could then be calculated based on the modeled wind and the cruise and metering-fix altitudes as shown in the following equation:

$$
\dot{h}_{g}=K_{g} \frac{W_{H, h_{c}}-W_{H, h_{\mathrm{MF}}}}{H_{c}-H_{\mathrm{MF}}} \quad[\mathrm{ft} / \mathrm{sec}]
$$

## Approximation of True Airspeed

It is necessary to determine true airspeed from
both Mach number and calibrated airspeed, as required for each path segment, so that a head-wind component can be added to obtain ground speed for time calculations. True airspeed, as a function of Mach number and static air temperature $T_{\mathrm{st}, H}$, was defined by the following equation (ref. 4):

$$
\mathrm{TAS}=38.96 M\left(T_{\mathrm{st}, H}\right)^{1 / 2} \quad[\mathrm{knots}]
$$

True airspeed, as a function of calibrated airspeed and altitude $h$, was approximated with the following empirical equation:

$$
\mathrm{TAS}=\frac{\mathrm{IAS}}{1-\left(0.12 \times 10^{-4}\right) h} \quad[\mathrm{knots}]
$$

where

$$
\begin{align*}
h & \leq 42000 & & {[\mathrm{ft}] }  \tag{ft}\\
220 & \leq \text { IAS } \leq 360 & & {[\text { knots }] }
\end{align*}
$$

## Wind Modeling Technique

A two-component linear wind model was used to represent the wind speed and the wind direction as functions of altitude. The coefficients of the wind model were computed via a linear regression analysis of data from winds-aloft reports and forecasts in the descent area. Winds-aloft data for the linear regression analysis were inserted through the calculator keyboard in a format similar to that used in standard aviation winds-aloft forecasts. Although wind speed and direction from only two altitudes were required to define a wind model, the pilot could choose to insert additional wind data based on both forecasts and pilot reports.

The magnitude of the wind speed and the direction of the wind defined by the linear wind model were computed for each segment of the profile based on the middle altitude of each segment. The following equations were used for these computations:

$$
\begin{aligned}
& S_{w, h}=\frac{d S_{w}}{d H} h+S_{w, s} \\
& D_{w, h}=\frac{d D_{w}}{d H} h+D_{w, s}
\end{aligned}
$$

A head-wind component for each segment was computed automatically during the profile computations by multiplying the wind speed with the cosine of the angle between the airplane ground track and the wind direction. A head-wind-component correction factor, based on the actual winds encountered during cruise flight, could also be added to the wind model if the pilot determined it was necessary. The correction factor was obtained by first computing the term $W_{c}$, which was the difference between the actual
ground speed along the cruise segment (computed by the pilot) and the predicted ground speed based on the modeled winds and cruise Mach number. This term was assumed to be proportional to altitude and decreased linearly to 0 at sea level $(h=0)$. The corrected head-wind component $W_{H, h}$ used in the profile computations was defined by the following equation:

$$
W_{H, h}=S_{w, h} \cos \left(D_{w, h}-\text { TRK }\right)+\left(h / h_{c}\right) W_{c} \quad[\text { knots }]
$$

where

$$
\mathrm{TRK}=\mathrm{CRS}-\mathrm{VAR}
$$

## Compensation for Effects of Nonstandard Atmospheric Temperature

Various flight instruments, including the Mach meter and the altimeter, are designed to display true indications in a standard atmosphere. However, standard atmospheric conditions are rarely encountered. This results in slight errors in speed and indicated altitude. The profile descent algorithm compensates for nonstandard temperatures as they affect the Mach number calculations and altimeter indications.

Nonstandard temperatures are computed by the algorithm based on a standard atmospheric temperature model with a bias correction based on the difference between the actual and the standard temperatures. The standard-temperature model is a twosegment linear profile defined as a function of altitude. This temperature model uses a slope equal to a temperature lapse rate of $-1.978 \times 10^{-3}{ }^{\circ} \mathrm{C} / \mathrm{ft}$ for altitudes below the tropopause ( $H \leq 36152 \mathrm{ft}$ ). At higher altitudes ( $H>36152 \mathrm{ft}$ ), it was assumed that flight was being conducted within the tropopause where temperature remains constant with changes in altitude. The standard-temperature model is represented mathematically as

$$
\begin{equation*}
T_{\mathrm{ISA}, H}=216.65 \tag{K}
\end{equation*}
$$

where $H>36152 \mathrm{ft}$ and as
$T_{\text {ISA }, H}=216.65+\left(1.978 \times 10^{-3}\right)(36152-H)$
where $H \leq 36152 \mathrm{ft}$.
The following bias correction $\Delta T$, representing the difference between the actual temperature measured at cruise altitude $T_{c}$ and the standard temperature for cruise altitude $T_{\text {ISA }, c}$, is added to the standard atmospheric temperature profile to define static temperature $T_{\mathrm{st}, H}$ completely at any altitude $H$ as follows:

$$
\Delta T=T_{c}-T_{\mathrm{ISA}, c}
$$

$$
\begin{equation*}
T_{\mathrm{st}, H}=T_{\mathrm{TSA}, H}+\Delta T \tag{K}
\end{equation*}
$$

The static temperature is then used for conversion of Mach number to true airspeed.

Pressure altitudes $H$ used to define the end points of each segment are corrected to approximate geopotential altitudes by multiplying the pressure altitude by a temperature ratio of nonstandard and standard sea-level temperatures (ref. 6) as follows:

$$
h=H\left(T_{o}^{\prime} / T_{o}\right) \quad[\mathrm{ft}]
$$

The standard sea-level air temperature $T_{o}$ is 288.15 K ; the nonstandard sea-level air temperature $T_{o}^{\prime}$ is computed from the static temperature model for $H=0$.

## Computations of Descent Path

The point where the pilot is to reduce power to idle thrust to start the descent was defined by summing the distances required to fly segments 1 through 6. Each segment length was determined by first computing the required time to traverse the segment and then multiplying by the average ground speed computed for the segment. Times for the levelflight segments requiring airspeed or Mach reductions were determined by dividing the required speed change by the deceleration capability of the airplane. Times for the path segments requiring descents were determined from equations derived by integrating the equation of the vertical-speed model over the altitude change required. The average ground speed at which the airplane was to fly each segment was determined by summing the computed true airspeed and the head-wind component evaluated for each segment.

The cruise segment (segment 7) at level flight and constant Mach number had no influence on the location of the point where idle thrust was to begin. This segment was significant only during the timemetered mode and was used for the calculations to satisfy the time constraints. Segments 1 and 2 were computed only if the ATC-imposed limit of 250 knots indicated airspeed for flight below 10000 ft MSL was applicable. The details of these calculations are presented in the following paragraphs.

M/IAS transition altitude. As the airplane descends at a constant Mach number, the indicated airspeed increases because of an increase in the air pressure. The altitude at which the desired descent airspeed is obtained is called the $M /$ IAS transition altitude and defines the point at which the constant Mach segment ends and the constant IAS segment begins. The general equation for this transition altitude was determined by equating true airspeed
as a function of indicated airspeed and altitude, with true airspeed as a function of Mach number and altitude. Solving for altitude results in the following equation to define the altitude for transition of Mach number to indicated airspeed:

$$
\begin{align*}
h_{\mathrm{XO}}= & 1.77675 \times 10^{5}-\left[8.90046 \times 10^{9}\right. \\
& \left.+\left(3.42936 \times 10^{7}\right) \frac{\mathrm{IAS}_{d}}{M_{d}}\right]^{1 / 2} \tag{ft}
\end{align*}
$$

Segment 1. Path segment 1 is a level-flight segment on which the airplane is slowed from an indicated airspeed of 250 knots to the metering-fix crossing speed. If the metering-fix crossing speed is less than or equal to 250 knots, or if the metering-fix altitude is equal to or greater than 10000 ft MSL, this segment is not computed. The equations for time and length in segment 1 are, respectively,

$$
\Delta t_{1}=\frac{\mathrm{IAS}_{\mathrm{MF}}-250}{\ddot{x}\left[1-\left(0.12 \times 10^{-4}\right) h_{\mathrm{MF}}\right]} \quad[\mathrm{sec}]
$$

where $\ddot{x}=-1.3$ knots $/ \mathrm{sec}$, and

$$
\Delta l_{1}=\left[\frac{\left(\mathrm{IAS}_{\mathrm{MF}}+250\right) / 2}{1-\left(0.12 \times 10^{-4}\right) h_{\mathrm{MF}}}-W_{H, h_{\mathrm{MF}}}\right] \frac{\Delta t_{1}}{3600} \quad[\mathrm{n} . \mathrm{mi} .]
$$

Segment 2. Segment 2 is an idle-thrust descent flown at a constant 250 knots from 10000 ft MSL to the metering-fix altitude. Segment 2 is not computed if the metering-fix altitude is equal to or greater than 10000 ft MSL or if the descent speed $\mathrm{IAS}_{d}$ flown on segment 4 is 250 knots or less. The equations for time and length in segment 2 are, respectively,

$$
\begin{equation*}
\Delta t_{2}=\frac{1}{C_{I}} \ln \left[\frac{h_{\mathrm{MF}} C_{I}+K_{\mathrm{gw}}\left(K_{\dot{h}, 250}+\dot{h}_{g}\right)}{10000 C_{I} \frac{T_{o}^{\prime}}{T_{o}}+K_{\mathrm{gw}}\left(K_{\dot{h}, 250}+\dot{h}_{g}\right)}\right] \tag{sec}
\end{equation*}
$$

where

$$
\begin{aligned}
K_{\dot{h}, 250} & =A_{I} \mathrm{e}^{250 B_{I}} & & {[\mathrm{ft} / \mathrm{sec}] } \\
A_{I} & =-3.07783 & & {[\mathrm{ft} / \mathrm{sec}] } \\
B_{I} & =8.158681 \times 10^{-3} & & {\left[\mathrm{knots}^{-1}\right] } \\
C_{I} & =-3.5 \times 10^{-4} & & {\left[\mathrm{sec}^{-\mathbf{1}}\right] }
\end{aligned}
$$

and
$\Delta l_{2}=\left[\frac{250}{1-\left(0.12 \times 10^{-4}\right) h}-W_{H, h}\right] \frac{\Delta t_{2}}{3600} \quad[$ n.mi.]
where

$$
\begin{equation*}
h=\left[\left(10000+H_{\mathrm{MF}}\right) T_{o}^{\prime} / T_{o}\right] / 2 \tag{ft}
\end{equation*}
$$

Segment 3. Segment 3 is a level-flight segment on which the airplane is slowed from the descent speed $\mathrm{IAS}_{d}$ to the metering-fix crossing speed (or 250 knots if segments 1 and 2 are computed). The equations for time and length in segment 3 are, respectively,

$$
\Delta t_{3}=\frac{\mathrm{IAS}_{\mathrm{MF}}-\mathrm{IAS}_{d}}{\ddot{x}\left[1-\left[0.12 \times 10^{-4}\right) h\right]} \quad[\mathrm{sec}]
$$

where $\ddot{x}=-1.3$ knots $/ \mathrm{sec}$, and

$$
\Delta l_{3}=\left[\frac{\left(\mathrm{IAS}_{d}+\mathrm{IAS}_{\mathrm{MF}}\right) / 2}{1-\left(0.12 \times 10^{-4}\right) h}-W_{H, h}\right] \frac{\Delta t_{3}}{3600} \quad[\mathrm{n} . \mathrm{mi} .]
$$

where
$h=\left\{\begin{array}{lll}h_{\mathrm{MF}} & {[\mathrm{ft}]} & \text { (if segments } 1 \text { and } 2 \text { are not computed) } \\ 10000 \frac{T_{o}^{\prime}}{T_{o}} & {[\mathrm{ft}]} & \text { (otherwise) }\end{array}\right.$

Segment 4. Segment 4 is an idle-thrust descent flown at a constant indicated airspeed $\mathrm{IAS}_{d}$. The descent begins at the transition altitude $h_{\mathrm{XO}}$ and ends at the metering-fix altitude (or at 10000 ft MSL , if segments 1 and 2 are computed). The equations for time and length of segment 4 are, respectively,
$\Delta t_{4}=\frac{1}{C_{I}} \ln \left[\frac{C_{I} H_{\mathrm{bod}} \frac{T_{o}^{\prime}}{T_{o}}+K_{\mathrm{gw}}\left(A_{I} \mathrm{e}^{B_{I} \mathrm{IAS}_{d}}+\dot{h}_{g}\right)}{C_{I} H_{\mathrm{XO}} \frac{T_{o}^{\prime}}{T_{o}^{\prime}}+K_{\mathrm{gw}}\left(A_{I} \mathrm{e}^{B_{I} I \mathrm{IA}_{d}}+\dot{h}_{g}\right)}\right] \quad[\mathrm{sec}]$
where

$$
H_{\mathrm{bod}}= \begin{cases}10000 & {[\mathrm{ft}]} \\ H_{\mathrm{MF}} & \text { (if segments } 1 \text { and } 2 \text { are computed) } \\ \text { (otherwise) }\end{cases}
$$

and
$\Delta l_{4}=\left[\frac{\mathrm{IAS}_{d}}{1-\left(0.12 \times 10^{-4}\right) h}-W_{H, h}\right] \frac{\Delta t_{4}}{3600} \quad[$ n.mi. $]$
where

$$
\begin{equation*}
h=\left[\left(H_{\mathrm{XO}}+H_{\mathrm{bod}}\right) T_{o}^{\prime} / T_{o}\right] / 2 \tag{ft}
\end{equation*}
$$

Segment 5. Segment 5 is a constant Mach descent flown at idle-thrust power settings. This segment begins at cruise altitude $h_{c}$ and ends when IAS $_{d}$ is attained at the transition altitude $h_{\mathrm{XO}}$. The
equations for time and length of segment 5 are, respectively,

$$
\begin{aligned}
\Delta t_{5}= & \frac{2 C_{M}}{K_{\mathrm{gw}}}\left\{\left(\frac{H_{c} \frac{T_{o}^{\prime}}{T_{o}}-c_{1}}{C_{M}}\right)^{1 / 2}-\dot{h}_{g}-K_{\dot{h}, \mathrm{XO}}\right. \\
& \left.-\dot{h}_{g} \ln \left[\frac{K_{\dot{h}, \mathrm{XO}}}{\left(\frac{H_{c} \frac{T_{o}^{\prime}}{T_{o}}-c_{1}}{C_{M}}\right)^{1 / 2}-\dot{h}_{g}}\right]\right\} \quad[\mathrm{sec}]
\end{aligned}
$$

where

$$
\begin{aligned}
& K_{\dot{h}, \mathrm{XO}}=\left(\frac{H_{\mathrm{XO}} \frac{T_{o}^{\prime}}{T_{o}}-c_{\mathbf{1}}}{C_{M}}\right)^{1 / 2}-\dot{h}_{g} \\
& c_{1}=M_{d} A_{M}+B_{M}
\end{aligned} \quad[\mathrm{ft}] \quad\left[\begin{array}{ll}
A_{M} & =25750 \\
B_{M} & =22167 \\
C_{M} & =-1.85
\end{array} \quad[\mathrm{ft}] \quad\left[\mathrm{sec}^{2} / \mathrm{ft}\right] \quad .\right.
$$

and

$$
\Delta l_{5}=\left[38.96\left(T_{\mathrm{st}, 5}\right)^{1 / 2} M_{d}-W_{H, h}\right] \frac{\Delta t_{5}}{3600} \quad[\mathrm{n} . \mathrm{mi} .]
$$

where

$$
\begin{equation*}
h=\left[\left(H_{c}+H_{\mathrm{XO}}\right) T_{o}^{\prime} / T_{o}\right] / 2 \quad[\mathrm{ft} \tag{ft}
\end{equation*}
$$

The static temperature $T_{\mathrm{st}, 5}$ and the head-wind component are evaluated at the average altitude between the cruise and transition altitudes.

Segment 6. Segment 6 is a level-flight speed change from the cruise Mach number to the descent Mach number. If the cruise and descent Mach numbers are the same, this segment is not computed. The equations for time and length of segment 6 are, respectively,

$$
\Delta t_{6}=38.96\left(T_{\mathrm{st}, c}\right)^{1 / 2} \frac{M_{d}-M_{c}}{\ddot{x}} \quad[\mathrm{sec}]
$$

where $\ddot{x}=-1.3$ knots $/ \mathrm{sec}$, and

$$
\Delta l_{6}=\left[38.96\left(T_{\mathrm{st}, c}\right)^{1 / 2} \frac{M_{c}+M_{d}}{2}-W_{H, h_{c}}\right] \frac{\Delta t_{6}}{3600} \quad[\mathrm{n} . \mathrm{mi} .]
$$

Segment 7. Segment 7 is the remaining path between the entry fix and the beginning of segment 6 . The length of segment 7 is the difference between the total distance between the entry fix and metering fix
$l_{t}$ and the sum of the distances of the remaining six segments. The length is given as follows:

$$
\Delta l_{7}=l_{t}-\sum_{j=1}^{6} \Delta l_{j} \quad[\text { n.mi. }]
$$

Segment 7 time $\Delta t_{7}$ is found by dividing the distance to be flown by the ground speed as follows:

$$
\Delta t_{7}=\frac{3600 \Delta l_{7}}{38.96\left(T_{\mathrm{st}, c}\right)^{1 / 2} M_{c}-W_{H, h_{c}}} \quad[\mathrm{sec}]
$$

## Input/Output Requirements

The data required for the profile descent equations are obtained from the preprogrammed calculator memory and from pilot entries through the keyboard shown in figure 11. Even though all the data necessary to compute the descent are entered prior to takeoff, these parameters may be updated during cruise to obtain more accurate results.

The wind data are entered through the keyboard and the wind model coefficients are automatically computed and stored in the proper memory locations. The wind data, correlated to altitude, are inserted in a data format similar to that found on an aviation weather forecast. To insert the wind data, the pilot must first push the key labeled "**", followed by the key labeled "*Wind". The display will request the altitude for the wind speed and direction data with the message " $H=$ ?FT". The altitude is keyed into the display and entered into memory by pushing the "New Entry" key. The calculator will then request the wind direction and speed with the message "DIR.SPD?". Wind direction and speed are keyed into the display and entered into memory by pushing the "New Entry" key. This process will be repeated until all wind data have been inserted in the calculator. The linear regression analysis will be completed after the pilot inserts a negative altitude to indicate that no more wind data will be inserted. The calculator will then display a "WIND IN" message.

The wind data used by the pilot to compute the wind model could contain some errors since that information is usually based on aviation forecasts that may not be current. A procedure was developed that allows the pilot to modify the wind model with a correction factor based on the difference between the computed and actual ground speeds along the cruise segment (segment 7). The computed ground speed used in the profile descent computations may be displayed by pushing the "*" key followed by the "*GSc" key. The difference between the displayed ground speed and the actual ground speed represents the wind modeling error along the magnetic course
of the airplane to the metering fix. If the ground speeds are different, the actual ground speed may be keyed into the display. Then, by pushing the "New Entry" key, the difference between the ground speeds is computed and stored in memory for use in subsequent descent and ground-speed computations.

The operational parameters affected by ATC constraints or pilot desires, and not accurately known until just prior to the start of descent, were designed to be single key inputs. To enter these data, the pilot presses the particular key dedicated to the parameter to be changed. After the key has been pressed, the display will show the name of the parameter and its current value that is stored in the calculator. Another numerical value may be keyed on the display and then stored in the proper memory location by simply pressing the "New Entry" key. If the current value shown is satisfactory, no more keyboard actions will be required for that parameter.

The operational parameters may be inserted in any order, or they may be changed at any time prior to initiating the descent calculations. When the magnitudes of the parameters are satisfactory to the pilot, the profile descent computations are initiated by pressing the "Profile" key. Computations typically require less than 2 min for completion in the time-metered mode of operation and approximately 25 sec in the nonmetered mode.

The operational parameters to be entered by the flight crew through the keyboard, as well as their symbology as presented on the keyboard and the display, are shown in table I.

If the descent speed schedule has been specified, a zero flag, indicating that the algorithm is in a speed mode, will be shown in the middle of the calculator display. While in the speed mode the entry-fix and metering-fix crossing times must remain unassigned. If these times are specified through the keyboard, the zero flag will not be shown and the algorithm will be in the time mode. While in the time mode the proper descent speed schedule will be computed and stored in the correct memory location for recall by the pilot.

When the computations are completed, the display will normally show the DME indication where thrust should be reduced to flight idle for the descent to the metering fix. It should be noted that the distance displayed is the point where an instantaneous thrust reduction and descent should be started. The pilot should start the descent 1 to 2 miles prior to the computed descent point to ensure that good passenger ride qualities are maintained during the transition to descent. If the assigned metering-fix crossing time cannot be attained in the time-metered mode because of airplane operational speed limitations, a
message will be displayed indicating the amount of time required to delay (hold) before starting the descent or the amount of time that the airplane will arrive late at the metering fix.

After the profile descent computations have been completed, the value of any operational parameters, including those required for input, may be displayed by pressing the particular designated key on the keyboard. Parameters that may be displayed after the descent computations, and their designated names, are shown in table II.

Open-loop guidance in the form of desired altitude as a function of distance along the profile may also be computed by the pilot. This is accomplished by keying a DME mileage indication into the display and pushing the "DME $\rightarrow H$ " key. The desired altitude corresponding to that distance will then be computed and displayed to the pilot. The "DME $\rightarrow H$ " feature may be used as guidance throughout the descent.

A program flow chart showing the steps used in the algorithm computations is included in appendix A. It is in a generalized format and may be used to aid in programming the algorithm on any computer. The actual program listing used with the $\mathrm{HP}-41 \mathrm{CV}$ calculator is included in appendix B .

## Flight Test Objectives

The flight tests consisted of two phases: (1) the performance-model validation phase, and (2) the operational evaluation phase. The objectives of the performance-model validation phase were to document the accuracy of the vertical performance model programmed into the calculator and to investigate the effect of variations in pilot technique on the arrival accuracy at the metering fix. These objectives were achieved by evaluating flight data in the form of time, speed, altitude, and DME indications. These data were recorded with a portable voice recorder on the flight segment between the entry fix and the metering fix.

The objectives of the operational evaluation phase were to determine if the concept of providing openloop descent guidance with a small, hand-held electronic computing device was acceptable to the pilot and if the concept was operationally feasible in an airline cockpit environment. These objectives were achieved by using quantitative data as recorded in the performance-model validation phase and subjective data in the form of pilot comments and test observer notes.

## Description of Airplane and Cockpit Instrumentation

The airplane type used during the flight tests
was a McDonnell Douglas DC-10-10 wide-body trijet commercial transport configured to carry 254 passengers. Flight tests were conducted on various DC-10-10 airplanes within the United Airlines fleet. The actual airplanes used in the tests were arbitrarily chosen. Even though all the DC-10 airplanes in the United Airlines fleet are powered by the same engines, rated at 39300 lb takeoff thrust, certified maximum takeoff weight varied between 410000 and 430000 lb . In addition, some of the test airplanes had been aerodynamically modified to reduce drag.

Specific flight instruments used by the pilot to fly the descent included an airspeed indicator and Mach number meter combined in one instrument, an altimeter, and a digital DME indicator. The airspeed indicator, Mach meter, and altimeter were driven by an air data computer that corrected pitot-static system inputs for sensor-position error and angle-ofattack effects. Static air temperature was also computed by the air data computer and displayed digitally on the instrument panel. DME indications were displayed digitally to tenths of a nautical mile in both upper left- and right-hand corners of the horizontal situation indicator. If the inertial navigation system were used, distance to, or from, the next way point would be displayed in place of the DME indications.

## Data Recording

A portable voice recorder and conventional flight instruments were used to collect data for descent performance and acceleration modeling. Altitude, Mach number, static air temperature, and DME readings were recorded at altitude increments of approximately 500 ft during the descents and at airspeed increments of approximately 10 knots during the idle-thrust, level-flight speed reductions. A stop watch was used to correlate the data with time during postflight analysis.

When the calculator was used to provide descent guidance, the contents of the calculator memory were printed with a portable printer after completing each descent. These data included all parameters inserted by the pilots and the results of intermediate computations that allowed reconstruction of the guidance computations for postflight analysis.

Flight notes were also recorded by the test conductor during the descents. These notes contained the resulting time and DME indication at which the metering-fix crossing altitude and airspeed were actually obtained, ATC instructions related to the descent, pilot comments, and other observations pertinent to the tests (i.e., turbulence, wind shear, and variations in pilot technique).

## Test Procedure

Performance-model validation tests. During the model validation flight tests, the calculator inputs were made by either the flight crew or the NASA test engineers. The tests were conducted on selected line flights and coordinated with ATC in an effort to yield, to the extent possible, uninterrupted descents. The pilot was asked to fly a specified $M$ /IAS speed schedule as accurately as possible, beginning the descent at the DME indication computed by the calculator.

Pilot-evaluation tests. During the pilot-evaluation tests, the subject pilots were given a briefing on the use of the descent calculator prior to the flight tests. During the tests, the subject pilots were asked to use the calculator in whatever manner they felt most useful. The NASA test engineers observed how the calculator was used and answered any questions about its use. The same subject pilots were used on 2 to 3 consecutive days on regular line flights to various large cities (Chicago, Denver, Boston, Newark, and Windsor Locks). Four to six descents were flown per subject pilot during both peak and off-peak traffic periods. There was no special ATC coordination during these tests. However, on some flights into Denver, a metering-fix crossing time (computed by the Denver Air Route Traffic Control Center enroute metering program) was given to the pilot by ATC. The metering-fix crossing time is normally retained for use by the ATC controller. When each subject pilot completed his series of descents, he was interviewed to obtain a subjective evaluation of the descent calculator.

## Results and Discussion

## Validation and Evaluation Criteria

The results of the performance-model validation and pilot-evaluation tests, as well as the results of a parametric sensitivity analysis conducted on the algorithm, were quantified in terms of the errors in time, altitude, and airspeed when the airplane crossed the metering fix and in terms of the time and distance required to achieve the desired meteringfix crossing altitude and airspeed. The results of the subjective evaluations by the subject pilots were summarized and are presented in a later section.

## Parametric Sensitivity Analysis

A parametric sensitivity analysis was conducted to determine the effects that uncertainties in the magnitudes of the operational parameters input for the descent computations would have upon the time
and distance predictions for crossing the metering fix. This analysis was conducted by comparing the output (i.e., time and distance predictions) of the descent computations for a nominal case with the output of a descent profile constructed to reflect the flight path resulting from an off-nominal flight condition. Each off-nominal descent was constructed by using the predicted time and distance computed with the same inputs used for the nominal case, except for the specific input parameter to be examined. The descent profile computed for the offnominal case was shifted so that the idle-thrust descent point computed for the off-nominal case corresponded to the DME indication computed for the nominal case. Then, the resulting time and airspeed errors for crossing the metering fix were calculated. The magnitude of each parameter examined in the sensitivity analysis was varied throughout a range of values that could typically be encountered.

Nominal case. The magnitudes of the parameters used to compute the nominal case were as follows:

Cruise altitude $=37000 \mathrm{ft}$
Cruise Mach number $=0.830$
Airplane gross weight $=304000 \mathrm{lb}$
Outside static air temperature $=-56.5^{\circ} \mathrm{C}$ (ISA at 37000 ft )

Descent Mach number $=0.830$
Descent airspeed $=280$ knots
Metering-fix crossing altitude $=8000 \mathrm{ft}$
Metering-fix crossing speed $=230$ knots
No wind
Entry-fix location $=100 \mathrm{n} . \mathrm{mi}$. from metering fix
The following results were obtained from the profile descent computations by using the nominal inputs. Thrust was to be reduced to flight idle to start the descent 83.0 n .mi. from the metering fix. A constant Mach number of 0.830 was to be maintained during the descent until an airspeed of 280 knots was obtained. The 280 -knot airspeed was to be maintained until reaching an altitude of 10000 ft MSL. At 10000 ft MSL, the airplane was to be slowed to 250 knots in level flight. After obtaining 250 knots, the descent was to be continued to 8000 ft MSL at which point the airplane would be flown at a constant altitude and slowed to 230 knots airspeed. The metering fix would be crossed as 230 knots was obtained. The time to fly from the idle-thrust point to the metering fix would be 802 sec .

The following parameters were varied in the sensitivity analysis: static air temperature, gross weight, Mach number, descent airspeed, wind magnitude, and wind gradient. The results of the sensitivity
analysis, shown in figure 12 as plots of altitude error, airspeed error, and time error that would result when the airplane crossed the metering fix, are shown for the range of values computed for each parameter. The distance prior to or past the metering fix where the desired metering-fix altitude and airspeed were attained is also plotted.

Static-air-temperature sensitivity. Static air temperature is used in the descent computations to convert pressure altitude to an approximate geopotential altitude and to compute true airspeed (in knots) from Mach number. The effects of static-air-temperature error on the descent profile were calculated by biasing the nominal temperature profile through a range from $10^{\circ} \mathrm{C}$ warmer to $10^{\circ} \mathrm{C}$ colder than standard. Temperatures warmer than standard resulted in an increase in both geopotential altitudes and true airspeed; temperatures colder than standard resulted in decreases. The metering-fix crossing errors resulting from temperature errors are plotted in figure 12(a).

With a temperature profile $5^{\circ} \mathrm{C}$ warmer than standard, the increased altitudes and airspeeds would cause the metering fix to be crossed 9.8 sec early, at an airspeed 20 knots faster than desired. The desired crossing airspeed and altitude would be attained 1.2 n.mi. past the metering fix. For a temperature profile $5^{\circ} \mathrm{C}$ colder than standard, the desired crossing speed would be attained $1.1 \mathrm{n} . \mathrm{mi}$. prior to the metering fix. The fix would be crossed 9.0 sec later than predicted. Although no altitude and airspeed error would occur with the colder temperature, extra fuel would be used to maintain altitude and airspeed.

Gross-weight sensitivity. The effects of grossweight variations on the descent profile were calculated for a weight range from 270000 to 350000 lb . These weights represented the minimum and maximum landing gross weights, respectively, that would likely be encountered in routine operations. The metering-fix crossing errors resulting from variations in gross-weight errors are plotted in figure $12(\mathrm{~b})$.

The descent parameter most significantly affected by gross-weight variations was the point at which thrust should be retarded to flight idle for beginning the descent. An airplane that was 3.3 percent ( 10000 lb ) heavier than that for the nominal case ( 304000 lb ) would require a path $2.6 \mathrm{n} . \mathrm{mi}$. longer than that predicted for the descent, since it must be flown at a slightly higher lift/drag ratio (same indicated airspeeds) that would result in a shallower descent angle. The metering fix would be crossed 14.4 sec earlier and 20 knots faster than desired. The desired airspeed would be achieved $2.6 \mathrm{n} . \mathrm{mi}$. past the fix. For the $10000-\mathrm{lb}$ lighter-than-nominal case, the
desired crossing speed and altitude would be achieved 2.5 n.mi. prior to crossing the metering fix. This would result in an arrival-time error of 12.1 sec late. Extra fuel would be used to maintain altitude and airspeed.

Mach number sensitivity. The effects of Mach number variations on the descent profile were calculated for two separate scenarios. In the first scenario, the cruise Mach number was constant and equal to 0.83 and the descent Mach number was varied for a range between 0.74 and 0.86 . The metering-fix crossing errors resulting from these variations are plotted in figure 12(c). The variation in the descent Mach number yielded the least relative variation in the metering-fix crossing errors. This was due primarily to the relatively short duration of the Mach descent segment.

In the second scenario, both the cruise and the descent Mach number were biased by a 0.02 Mach increment faster and slower to represent an error in the Mach indicator on board the airplane. At the higher Mach number ( 0.85 ), the metering fix was crossed 8.9 sec early with an airspeed 15 knots higher than desired. The desired 230 -knot airspeed was attained $1.0 \mathrm{n} . \mathrm{mi}$. past the metering fix. At the lower Mach number ( 0.81 ), the metering fix was crossed 12.3 sec late. The desired airspeed and altitude were attained $1.2 \mathrm{n} . \mathrm{mi}$. prior to the metering fix and were maintained at a cost of increased fuel.

Descent airspeed sensitivity. The effects of descent airspeed variations on the descent profile were calculated for an airspeed range between 230 and 350 knots. Metering-fix crossing errors were plotted in figure 12(d) for variations of airspeed at altitudes above 10000 ft MSL. The distance error plot shows that variations in the descent airspeed resulted in corresponding changes in the distance required to attain the metering-fix crossing conditions. A $10-\mathrm{knot}$ speed increase ( 290 -knot descent airspeed) resulted in attaining the metering-fix conditions $3.0 \mathrm{n} . \mathrm{mi}$. prior to crossing the metering fix; a 10 -knot airspeed decrease, $2.9 \mathrm{n} . \mathrm{mi}$. past the fix. The time error to cross the metering fix, when flown at the slower airspeeds, also resulted in a corresponding increase. (A 270 -knot airspeed resulted in a 11.1 -sec late time error.) However, when airspeeds higher than the nominal speed were used, time error was not significantly changed. (A 290-knot airspeed resulted in a 2.5 -sec early time error.) This was caused by the fact that the metering-fix altitude and airspeed conditions were attained prior to crossing the metering fix. The additional time required to fly to the metering fix would then offset the time saved with the
higher descent speed and would eventually result in crossing the metering fix later than desired, as shown in figure $12(\mathrm{~d})$.

Wind modeling sensitivity. The effects of wind modeling errors were calculated first, by assuming a constant 20 -knot error in the head-wind component for the entire flight path between the entry fix and the metering fix. These calculations were then repeated by assuming a 20 -knot tail-wind error.

With an unknown $20-\mathrm{knot}$ head wind, the resulting ground speed would be decreased proportionately and would result in a crossing-time error of 76.8 sec later than predicted. The desired airspeed and altitude for crossing the metering fix would be attained $4.5 \mathrm{n} . \mathrm{mi}$. prior to the metering fix. Thrust would be required to maintain airspeed and altitude to the metering fix. With the 20 -knot tail wind, the metering fix would be crossed 58.9 sec earlier than predicted, but with an airspeed 20 knots faster than desired. The desired final airspeed and altitude would be achieved $4.4 \mathrm{n} . \mathrm{mi}$. past the metering fix.

It should be noted that the time error is accumulated both in the descent and cruise portions of flight between the entry and metering fixes. Hence, this time error will also be a function of distance between the fixes. If the time error incurred during the cruise portion is not considered, the time error for the head-wind case would be 71.1 sec late; and for the tail-wind case, 53.7 sec early.

The potential for significant errors occurring in wind modeling is high because of a constantly changing atmosphere and the vagaries of forecasting winds aloft. However, the wind modeling accuracy problem may be reduced if the pilot modifies the wind model based upon the winds actually encountered during the cruise portion of his flight through the cruise ground-speed function (*GSc key). By pushing the "*" and the "*GSc" keys, the ground speed will be computed for the cruise segment of flight based on the wind model in the calculator. If the computed and actual ground speeds are different, the actual ground speed can be entered into the calculator so that the wind model will be modified for subsequent computations. The wind model will be changed by summing an error component to the original wind model. The error component is equal to the difference in the computed and actual ground speeds, linearly decreased to 0 from cruise altitude to sea level.

Wind-gradient sensitivity. The effects of wind gradient (change in head-wind component due to a change in altitude) were calculated for a range of magnitudes between -4 knots $/ 1000 \mathrm{ft}$ (increasing head wind from cruise to the metering fix) and

4 knots/ 1000 ft (decreasing head wind from cruise to the metering fix). The wind gradients were input such that the average head wind from cruise to the metering fix was 0 . The results of these changes are shown in figure 12(e).

The effect of an increasing head wind on the airplane as it descends is to lengthen the distance required to attain the metering-fix crossing conditions. This results in an early arrival at the metering fix at a higher airspeed than desired. The increased distance to descend is due to the shallowing of the flight path angle that is required to maintain the desired descent airspeed.

## Performance-Model Validation Tests

The purpose of the initial flight testing and data analysis was to validate and refine the descent performance model used in the profile descent algorithm. Altitude, DME, airspeed, and temperature data were recorded continuously during the descent so that the flight profile could be reconstructed for postflight analysis. The same subject pilot flew each of the descents during this phase of the evaluation. The subject pilot also advised ATC that NASA fuel conservation tests were in progress and requested an unrestricted descent.

The criterion used to evaluate the performance that resulted with the open-loop guidance was the accuracy in terms of the descent time and the descent distance required to achieve the desired metering-fix altitude $h_{\mathrm{MF}}$ and airspeed IAS $\mathrm{MF}_{\mathrm{MF}}$ and the accuracy in terms of the resulting time error when the metering fix was crossed. These errors are presented in table III. The mean and standard deviation for these errors are presented at the bottom of the table. The origin and destination airports and the $M /$ IAS descent speed schedule for each test run are also shown in the table.

The time error associated with attaining the desired metering-fix airspeed and altitude was an indication of the accuracies with which the airplane descent performance data had been modeled and how closely the airplane had been flown on the predicted profile with the open-loop guidance. This particular form of time error was not affected by horizontal winds since it was a time associated with attaining airspeeds and altitudes that are air-mass referenced. However, wind-speed-gradient effects (the change in head-wind component due to a change in altitude) can have an effect upon this form of time error. Runs 10 and 16 are examples in which large time errors resulted in the presence of a known windspeed gradient recorded from airborne, inertial navigation system data. The mean and standard deviation of the time error to attain the desired airspeed
and altitude for the 19 test runs were 1.3 sec early and 22.4 sec , respectively, for descents lasting between 5 and 14.3 min . Based on these time errors, it was concluded that the airplane descent performance data were adequately modeled and that the time required to attain the desired airspeed and altitude could be adequately predicted with the openloop guidance provided by the Mach and airspeed indicators.

The distance error that resulted while attaining the desired metering-fix speed and altitude and the time error associated with crossing the metering fix were indicators of the same error components associated with the descent time error discussed previously and, in addition, indicators of the accuracy of wind modeling. The resulting mean and standard deviation for the distance error for these test runs were 2.3 n.mi. past the metering fix and 2.3 n.mi., respectively. The mean and standard deviation for the crossing-time error at the metering fix were 24.4 sec early and 28.3 sec , respectively. These errors are consistent (i.e., approximately $1 / 2 \mathrm{~min}$ is required to fly 2.3 n.mi. past the metering fix) and are referenced to Earth-fixed axes (i.e., the metering fix) and, as such, are affected by the wind. Since a much smaller time error to attain the desired altitude and airspeed (air-mass referenced) was achieved, it was concluded that the data inserted into the wind model may have been in error.

Experience obtained during the profile descent tests with the Advanced Transport Operating Systems TSRV Boeing 737 airplane (ref. 4) and with the NASA T-39A airplane (ref. 5) indicates that the linear wind model is sufficient to model the wind during the descent, provided that the model is based on reasonably accurate wind data. However, the windsaloft weather forecast may be based on observations more than 12 hr old and, as such, may be subject to some error.

## Variation in Pilot Technique

Variation in pilot technique in flying the descent can have an effect upon the crossing errors at the metering fix. Three phases of the descent were noted during these tests as having a significant impact on the crossing accuracy. They were the thrust reduction and the pitch rate used at the start of the descent, the pilot technique used to control the pitch attitude of the airplane during the constant Mach number descent segment, and the technique used to make the transition to the level flight on the speedreduction segments.

It was initially assumed that the pilot would reduce the thrust to flight idle with a rather short deliberate action. However, in the interest of passenger
comfort, the pilot would gradually reduce the throttles over a 10 - to 20 -sec interval and would slowly decrease the pitch attitude of the airplane to begin the descent. In addition, all three throttles were seldom reduced in unison to avoid the simultaneous shifting of engine bleed air valves which causes loud noises in the pressurization system. Because of the gradual thrust reduction, it was found that some anticipation of the idle-thrust point was required for the throttles to be at idle at that point. The amount of lead distance required varied with the speed with which the pilot reduced the throttle. The method utilized by most pilots was to gradually reduce power to idle thrust on the left and right engines about 2 n .mi. prior to the computed idle-thrust descent point. After the left and right engines were reduced to flight idle, the center engine would be similarly reduced.

The second phase of the descent in which the metering-fix crossing accuracy could be affected was in the constant Mach number descent segment. To maintain a constant Mach number during an idlethrust descent, the pitch attitude of the airplane must be continuously decreased as altitude is lost. As a result, cabin deck angles may become excessive for passenger comfort during high-speed descents. Some pilots approximated the constant Mach descent segment by flying a constant vertical-speed descent instead. If the airspeed that will be used in the constant airspeed descent segment is low, then the Mach segment will be short and the resulting errors small. If the constant airspeed segment is to be flown at higher airspeeds, the Mach segment will be longer and greater errors will accrue. As an example, from a cruise altitude of 37000 ft , with a descent speed schedule of $0.830 / 280$ knots, only 2600 ft must be lost during the constant Mach segment; with a descent speed schedule of $0.830 / 340$ knots, 10800 ft must be lost.

The last phases of the descent, in which piloting technique affected the vertical performance, were the level-flight deceleration segments at 10000 ft and at the bottom of the descent. If the pilot gradually decreased the descent rate, by beginning 1000 to 2000 ft above the level-flight altitude to achieve a gradual transition to level flight, the distance required to decelerate from the descent speed to the metering-fix crossing speed was longer than if the pilot chose to descend slightly below the level-flight altitude and gradually climb back up to it. The pilot could use either of these techniques to null distance or time error.

## Pilot-Evaluation Test Phase

Five subject pilots were briefed on the use of the calculator during the pilot-evaluation phase of
the tests. Each pilot participated in four, or more descents while using the calculator. Twenty-two descents were completed during the pilot-evaluation phase of the tests.

The flight test results achieved during this phase are presented in table IV. These results include the metering-fix crossing errors that were defined as the difference between the actual and the computed crossing time, altitude, and indicated airspeed. Also tabulated were the origin and destination airports, the airport arrival time, if ATC altered or constrained the descent clearance, if the calculator was unusable, if the pilot used an $M /$ IAS speed schedule or the DME $\rightarrow H$ feature for descent guidance, and remarks about any anomalies incurred during the descent.

Each subject pilot followed the same general procedures when using the calculator. Preliminary inputs to the calculator were made prior to takeoff from a computerized flight plan normally provided by the company flight dispatcher. Data from the computerized flight plan input to the calculator included an estimated gross weight, cruise altitude, cruise Mach number, winds-aloft forecast, and outside air temperature. The pilot would estimate altitude and airspeeds to cross the metering fix based on previous flight experience at the destination airport. The pilot would then compute a preliminary descent profile. He would update the input parameters later in the flight and recompute the descent profile as required. The final descent profile would typically be computed 150 to $200 \mathrm{n} . \mathrm{mi}$. from the metering fix. However, on several occasions during this phase of the test, to satisfy ATC constraints, the pilot was required to reprogram the calculator just prior to the top-of-descent point and during the descent.

The mean and standard deviation of the crossing errors at the metering fix are shown at the bottom of table IV. Except for the mean time error at the metering fix, the crossing errors were larger than those obtained during the performance-model validation tests because of ATC clearance changes. Only 6 of the 22 runs were completed without receiving an amended ATC clearance. Amended clearances received during the descent were more difficult to modify on the calculator than those received prior to the descent. On four of the descents, the calculator was not used at all because of numerous ATC clearance changes received throughout the descent. On five of the descents, the guidance provided by the calculator was used for only a portion of the descent because of ATC requests for spacing and sequencing.

Data in the calculator were modified by the pilot just prior to the descent to accommodate amended ATC clearances on six of the descents. However,
some of the amended clearances did not require data modification. As an example, if the pilot was told to descend early to an intermediate altitude, the pilot would descend to, and remain at, the intermediate altitude until he had reacquired the original profile as determined from the $\mathrm{DME} \rightarrow H$ feature. The idlethrust descent would then be continued by using the guidance provided by the calculator.

If an amended ATC clearance were received in enough time prior to the top-of-descent point, the flight crew could modify the data in the calculator and then the full benefits of fuel conservation from use of the guidance could be obtained. Amended clearances received after the descent has begun may, or may not, preclude the use of the guidance provided by the calculator. However, amended clearances received after the descent has begun will almost always result in more fuel being used. Amended ATC clearances may be necessary for safe and expeditious movement of airplanes in a very dynamic environment. It was noted, however, that an amended clearance was received on only one of the descents during the performance-model validation tests during which the pilot told ATC that fuel-conservation tests were in progress.

The time mode of the calculator was used on only three of the descents during the pilot-evaluation test phase. All these descents were into Denver since this was the only destination terminal of the various flights actively using time-based metering. On these flights the pilot input the actual time that the airplane crossed an arbitrary entry fix (typically about $130 \mathrm{n} . \mathrm{mi}$. from the metering fix) and the metering-fix crossing time assigned by ATC. The pilot would then start the computation of the descent profile. The computations typically required about 2 min to complete.

On two of the three time-metered flights, the crossing-time accuracy was within $\pm 6 \mathrm{sec}$. On flight 20, however, the pilot started the descent late and used a higher descent speed in an attempt to reacquire the computed descent profile. Even though the altitude error was nulled, the metering fix was crossed 50 sec earlier than desired and 35 knots faster than planned.

On seven descents, the pilot used the $M /$ IAS speed schedule provided by the calculator as the primary descent guidance. On 11 of the descents, the altitude profile generated by the calculator through the DME $\rightarrow H$ function was used as the primary descent guidance. When the DME $\rightarrow H$ function was used, one of the nonflying crew would operate the calculator and call out the altitudes corresponding to upcoming DME indications. When this form of guidance was used instead of maintaining an $M$ /IAS
descent speed schedule, slightly earlier arrival times at the metering fix resulted. These results occurred because the airplane was typically initially high on the computed vertical profile since the pilots would make the transition from cruise flight into the descent slowly because of passenger ride-comfort considerations. As the pilot increased the vertical speed of the airplane to correct back to the vertical profile, a higher airspeed would result. If the airplane were below the computed profile, it would be expected that the metering fix would be crossed at a later time.

## Pilot Comments

Each pilot was interviewed at the conclusion of the series of descents to obtain a subjective evaluation of the descent calculator concept. All subject pilots in general derived the same consensus. A compendium of the pilot comments is as follows:

1. The work load required to operate the calculator was acceptable provided that major modifications to the data were not required during the descent. The pilots felt that the use of the calculator did not interfere with their normal flight duties or ever compromise safety.
2. All the pilots felt that some training period would be required to use the calculator operationally. All but one of the pilots felt that the calculator was much easier to learn to operate than an inertial navigation system.
3. Most of the pilots felt that they could plan the descent as well by using various rules of thumb for their standard airline-policy $M$ /IAS descent. However, for descents at other than standard $M$ /IAS speed schedules or for time-metered descents, the calculator was very useful.
4. All the pilots felt that the guidance provided by the DME $\rightarrow H$ function was very useful.
5. Even though no operational problems due to the computation speed were encountered during these tests, it was felt that the computation time should be reduced, especially for the time-metered computations.
6. Some of the pilots wanted to be able to recall the wind data that was input so that key-punching errors could be rechecked and corrected. With the present software configuration, wind inputs can be checked only prior to pushing the "New Entry" key.
7. All the pilots felt that the display was difficult to read at night in reduced lighting. This is typical of all liquid-crystal displays that are not backlighted.

## Suggested Modifications

During the flight tests, situations arose which highlighted areas in which design improvements to
the descent calculator could enhance its use. The following is a list of features which should be considered in any future design of a descent planning calculator:

1. The DME distance term presently used in the DME $\rightarrow H$ function represents the distance between the airplane and the DME ground station parallel to the Earth's surface. The DME indication in the airplane represents the distance directly between the airplane and the DME ground station (called slant range). The difference between the horizontal distance and the slant range distance should be included in the algorithm so as to yield a more accurate altitude computation.
2. The altimeter setting of the destination airport should be included in the calculator inputs so that a correction for the change in the indicated altitude can be accounted for when changing from a pressure altitude setting to a corrected altimeter setting.
3. The wind data used in generating the wind model should be stored so that they can be reviewed and modified.
4. The display should be lit so that it is readable in a reduced-lighting environment.
5. The computation time should be reduced. This could be accomplished by using a small portable computer rather than a programmable calculator to exercise the algorithm.
6. A function to compute the DME reading(s) corresponding to a given altitude should be included as an alternate form of guidance to the DME $\rightarrow H$ function (which provides the altitude corresponding to a given DME reading).
7. A function to compute the desired time corresponding to a DME distance input (similar to the DME $\rightarrow H$ function) should be included.
8. A revised temperature model that can account for nonstandard lapse rates and nonstandard troposphere altitudes may result in higher accuracy.

## Concluding Remarks

A simple, airborne, flight-management descent algorithm, designed to aid the pilot in planning a fuelefficient time-constrained descent, was programmed into a small programmable calculator. In a timemetered mode, the airborne algorithm computes the specific Mach number, airspeed, and point for the pilot to begin the descent to arrive at a metering fix at a predetermined airspeed, altitude, and time assigned by air traffic control (ATC). In the nonmetered mode, the algorithm computes the point to begin the descent based on the Mach and airspeed descent schedule input by the pilot.

Flight test data obtained in an airline operational environment during normally scheduled flights indicated that the time and distance to descend could be satisfactorily predicted with the use of relatively simple equations and airplane-performancecharacteristics modeling. The flight data also have shown that the descent profile could be satisfactorily flown with open-loop guidance provided by conventional cockpit Mach and airspeed indicators. However, pilot technique used to fly the descent can affect the accuracy of crossing the metering fix at the desired time.

The subject pilots commented that the use of the calculator to plan descents was particularly useful when nonstandard descent speeds were used or when the time to cross the metering fix had been assigned by ATC. They also found that the calculator did not interfere with their normal flight duties. Amended clearances received from ATC, after the descent had begun, precluded the use of the calculator for descent guidance when extensive modifications to the input data were required.

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TABLE I. OPERATIONAL PARAMETERS REQUIRED FOR DESCENT COMPUTATIONS

| Keyboard symbology | $\begin{gathered} \text { Display } \\ \text { symbology } \end{gathered}$ | Operational parameter (symbol used in equations) |
| :---: | :---: | :---: |
| Mc | Mc | Cruise Mach number ( $M_{c}$ ) |
| Hc | Hc | Cruise altitude ( $H_{c}$ ), ft |
| *GSc | GSc | Ground speed at cruise altitude (GSc), knots |
| Md | Md | Descent Mach number ( $M_{d}$ ) |
| IASd | IASd | Descent indicated airspeed ( $\mathrm{IAS}_{d}$ ), knots |
| Time MF | MF TM | Time assigned by ATC to cross metering fix ( $\mathrm{t}_{\mathrm{MF}}$ ), hr:min:sec |
| Time EF | EF TM | Time to cross entry fix ( $t_{\mathrm{EF}}$ ), hr:minisec |
| GW | GW | Airplane gross weight at top of descent (GW), lb |
| OAT | OAT | Static outside air temperature (OAT), ${ }^{\circ} \mathrm{C}$ |
| Metering Fix | H MF | Pressure altitude of metering fix ( $H_{\mathrm{MF}}$ ), ft |
| Metering Fix | IAS MF | Indicated airspeed to cross metering fix, ( IAS $_{\text {MF }}$ ), knots |
| Metering Fix | MF DME | DME indication defining metering-fix location ( $\mathrm{MF}_{\text {DME }}$ ), n.mi. |
| Entry Fix | EF DME | DME indication defining entry-fix location; this mileage must be relative to same DME station used to define metering fix ( $\mathrm{EF}_{\mathrm{DME}}$ ), n.mi. |
| Entry Fix | MAGCRS | Magnetic course from entry fix to metering fix (CRS), deg |
| Entry Fix | MAGVAR | Magnetic variation in descent area (VAR), deg |

TABLE II. PARAMETERS COMPUTED BY DESCENT COMPUTATIONS

| Keyboard symbology | $\begin{gathered} \text { Display } \\ \text { symbology } \end{gathered}$ | Operational parameter (symbol used in equations) |
| :---: | :---: | :---: |
| Md | Md | Descent Mach number required to satisfy entry-fix and metering-fix time constraints ( $M_{d}$ ) |
| IASd | IASd | Descent indicated airspeed required to satisfy entry-fix and metering-fix time constraints ( $\mathrm{IAS}_{d}$ ), knots |
| Idle DME | IDL DME | DME indication showing point where thrust should be reduced to flight idle (IDL ${ }_{D M E}$ ), n.mi. |
| 2nd push of Idle DME | IDL TM | Incremental time to fly between entry fix and metering fix ( $\Delta t_{\text {req }}$ ), sec |
| Late | LATE | Amount of time that airplane will be late in crossing metering fix, min:sec |
| Early | HOLD | Amount of time that airplane must delay before starting descent if crossing-time constraint at metering fix is to be satisfied, min:sec |

TABLE III. TIME AND DISTANCE ERRORS TO ATTAIN METERING-FIX CONDITIONS DURING PERFORMANCE-MODEL VALIDATION TESTS

|  | Airport, origin- | $M_{d} / \mathrm{IAS}_{d}$, | Des attain | distance <br> MF and mi. |  | Desce IAS | me to and $h_{M}$ sec |  |  | to cross ring fix, sec |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Run | destination | knots | Calculated | Actual | Error | Calculated | Actual | Error | Calculated | Actual | Error |
| 1 | ORD-DEN | 0.730/250 | 68.3 | 70.1 | -1.8 | 681 | 671 | 10 | 681 | 653 | 28 |
| 2 | DEN-ORD | .710/250 | 81.4 | 84.7 | -3.3 | 828 | 841 | -13 | 828 | 799 | 29 |
| 3 | ORD-DEN | .814/289 | 60.1 | 61.7 | -1.6 | 538 | 540 | -2 | 538 | 525 | 13 |
| 4 | ORD-DEN | .830/296 | 60.0 | 60.0 | 0 | 540 | 535 | 5 | 540 | 535 | 5 |
| 5 | DEN-ORD | .830/280 | 95.7 | 95.2 | . 5 | 867 | 836 | 31 | 867 | 844 | 23 |
| 6 | ORD-DEN | .834/350 | 34.5 | 37.4 | -2.9 | 300 | 298 | 2 | 300 | 273 | 27 |
| 7 | DEN-LAX | .830/280 | 83.0 | 83.8 | -. 8 | 779 | 758 | 21 | 779 | 796 | -17 |
| 8 | LAX-DEN | .830/280 | 62.2 | 64.6 | -2.4 | 526 | 540 | -14 | 526 | 517 | 9 |
| 9 | DEN-SFO | .820/280 | 70.8 | 76.0 | -5.2 | 698 | 725 | -27 | 698 | 660 | 38 |
| 10 | SFO-HNL | .820/280 | 73.0 | 80.8 | -7.8 | 716 | 752 | -36 | 716 | 671 | 45 |
| 11 | HNL-SFO | .820/280 | 97.2 | 97.8 | -. 6 | 898 | 860 | 38 | 898 | 852 | 46 |
| 12 | SFO-DEN | .830/280 | 69.6 | 72.4 | -2.8 | 567 | 605 | -38 | 567 | 567 | 0 |
| 13 | DEN-ORD | .820/280 | 73.9 | 78.4 | -4.5 | 608 | 606 | 2 | 608 | 561 | 47 |
| 14 | ORD-DEN | . $820 / 280$ | 61.1 | 59.0 | 2.1 | 555 | 565 | -10 | 555 | 591 | -36 |
| 15 | DEN-SFO | .830/280 | 76.2 | 78.4 | -2.2 | 692 | 700 | -8 | 692 | 674 | 18 |
| 16 | SFO--HNL | .800/280 | 55.3 | 59.9 | -4.6 | 645 | 601 | 44 | 645 | 545 | 100 |
| 17 | HNL-SFO | .830/320 | 70.2 | 71.9 | -1.7 | 708 | 706 | 2 | 708 | 683 | 25 |
| 18 | SFO-DEN | .830/280 | 65.4 | 69.0 | -3.6 | 569 | 560 | 9 | 569 | 526 | 43 |
| 19 | ORD-DEN | .830/280 | 64.9 | 65.9 | -1.0 | 599 | 590 | 9 | 599 | 579 | 20 |
| Mean <br> Standard deviation |  | . . . . | . . . . . |  | -2.3 |  |  | 1.3 | - . . . . . . . . |  | 24.4 |
|  |  |  | - . . . . |  | 2.3 |  |  | 22.4 |  |  | 28.3 |


| Run | Airport <br> origindestination | Local <br> airport <br> arrival <br> time, <br> $\mathrm{hr}:$ min | Metering-fix crossing errors |  |  | ATC <br> altered <br> descent | $\begin{array}{\|c} \text { Calculator } \\ \text { was } \\ \text { reprogrammed } \\ \hline \end{array}$ | Calculator was unusable | Descent was time metered | Primary guidance |  | Remarks |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  | Time $\begin{aligned} & \text { (Early }=+; \\ & \text { Late }=-) \end{aligned}$ | Indicated <br> airspeed $\text { (Fast }=+ \text {; }$ $\text { Slow }=-) \text {, }$ | Altitude $\begin{aligned} & \text { (High = +; } \\ & \text { Low }=- \text { ), } \end{aligned}$ |  |  |  |  |  |  |  |
|  |  |  | sec | knots | ft |  |  |  |  | M/IAS | DME $\rightarrow H$ |  |
| 1 | DEN-ORD | 4:04 p.m. | 85 | 40 | 0 |  |  |  |  |  | X | Started descent late ( $1.0 \mathrm{n} . \mathrm{mi}$. ); flew higher airspeed to reacquire vertical profile. |
| 2 | ORD-BOS | 8:17 p.m. | -6 | 15 | 0 |  |  |  |  |  | X |  |
| 3 | BOS-ORD | 9:10 a.m. | . 18 | 40 | 600 | X | X |  |  |  | X | ATC requested increase in airspeed; pilot maintained profile and added thrust; ATC later changed meteringfix altitude and location; calculator was reprogrammed accordingly. |
| 4 | ORD-DEN | 11:43 a.m. | 27 | 0 | 1400 | X |  |  |  |  | X | ATC clearance to descend was 3.4 n.mi. past computed idle DME point; airplane was also held at intermediate altitude for traffic before being cleared for profile descent; pilot flew at faster airspeed to reacquire vertical profile. |
| 5 <br> 6 | DEN-ORD | 4:04 p.m. |  |  |  | X |  | X |  |  |  | ATC changed clearance during descent too frequently to reprogram calculator. |
| 6 <br>  <br> 7 | ORD-BOS | 8:17 p.m. | 56 | 30 | 1200 | X | X |  |  |  | X | ATC requested early descent to intermediate altitude; calculator was reprogrammed to reflect change; new descent started $5.9 \mathrm{n} . \mathrm{mi}$. late; pilot flew at faster airspeed to reacquire vertical profile. |
| 7 | BOS-ORD | 9:10 a.m. |  |  |  | X |  | X |  |  |  | ATC changed clearance during descent too frequently to reprogram calculator. |


| Run | Airport origindestination | Local <br> airport <br> arrival <br> time, <br> $\mathrm{hr}: \mathrm{min}$ | Metering-fix crossing errors |  |  |  | $\begin{array}{\|c\|} \text { Calculator } \\ \text { was } \\ \text { reprogrammed } \\ \hline \end{array}$ | $\begin{gathered} \text { Calculator } \\ \text { was } \\ \text { unusable } \end{gathered}$ | Descent was time metered | Primary <br> guidance |  | Remarks |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  | $\begin{gathered} \text { Time } \\ \text { (Early }=+; \\ \text { Late }=-), \end{gathered}$ | $\begin{gathered} \hline \text { Indicated } \\ \text { airspeed } \\ \text { (Fast }=+; \\ \text { Slow }=- \text { ), } \end{gathered}$ | Altitude $\begin{aligned} & (\text { High }=+; \\ & \text { Low }=-), \end{aligned}$ |  |  |  |  |  |  |  |
|  |  |  | sec | knots | ft |  |  |  |  | M/IAS | DME $\rightarrow H$ |  |
| 8 | ORD-DEN | 11:43 a.m. | 6 | 5 | 1000 | X |  |  | X | X |  | ATC requested decrease in airspeed midway through descent; calculator was not used after speed reduction. |
| 9 | ORD-BOS | 2:44 p.m. | 33 | 30 | 1000 |  |  |  |  | X |  |  |
| 10 | BOS-ORD | 5:45 p.m. | -13 | 0 | 0 | X | X |  |  | X |  | ATC requested early descent to intermediate altitude; calculator was reprogrammed accordingly. |
| 11 | ORD-EWR | 2:40 p.m. | -52 | 0 | 0 |  |  |  |  | X |  | Experienced large change in head wind during last third of descent. |
| 12 | EWR-ORD | 5:40 p.m. | -21 | 0 | -1000 | X |  |  |  | X |  | ATC requested speed reduction and changed crossing restriction; pilot used speed brakes to increase descent rate. |
| 13 | ORD-DEN | 8:10 p.m. | -47 | 0 | 0 |  |  |  |  | X |  | Began descent off track because of ATC vectoring for spacing. |
| 14 | DEN-ORD | 6:54 p.m. | 13 | 5 | -1000 | X | X |  |  |  | X | ATC changed crossing restriction; calculator was reprogrammed; ATC requested speed increase. |
| 15 | BDL-ORD | 12:18 p.m. | 20 | 10 | 300 | X | X |  |  |  | X | ATC changed metering fix midway through descent; calculator was reprogrammed; new vertical profile was acquired. |
| 16 | ORD-DEN | 2:39 p.m. | 21 | -5 | 0 | X | X |  |  |  | X | ATC requested speed reduction; calculator was reprogrammed; pilot held at intermediate altitude and idle thrust to slow and acquire new vertical profile. |
| 17 | DEN-ORD | 10:50 p.m. |  |  |  | X |  | X |  |  |  | ATC changed clearance during descent too frequently to reprogram calculator. |

TABLE IV. Concluded

| Run | Airport <br> origin- <br> destination | Local <br> airport <br> arrival <br> time, <br> hr :min | Metering-fix crossing errors |  |  | ATC <br> altered <br> descent | $\begin{array}{\|c\|} \text { Calculator } \\ \text { was } \\ \text { reprogrammed } \\ \hline \end{array}$ | $\begin{array}{\|c} \text { Calculator } \\ \text { was } \\ \text { unusable } \\ \hline \end{array}$ | Descent was time metered | Primary guidance |  | Remarks |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  | Time$\begin{gathered} \text { (Early }=+; \\ \text { Late }=- \text { ) } \\ \text { sec } \end{gathered}$ | Indicated <br> airspeed <br> (Fast $=+$; <br> Slow $=-$ ), <br> knots | Altitude$\begin{gathered} (\text { High }=+; \\ \text { Low }=- \text { ), } \\ \mathrm{ft} \end{gathered}$ |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  | M/IAS | DME $\rightarrow H$ |  |
| 18 | ORD-DEN | 2:30 p.m. | -27 | 0 | 1100 | X |  |  |  |  | X | Descent was interrupted because of intermediate altitude restriction; thrust was added at intermediate altitude to maintain airspeed until cleared to continue descent. |
| 19 | DEN-ORD | 7:05 p.m. |  |  |  | X |  | X |  |  |  | ATC changed clearance during descent too frequently to reprogram calculator. |
| 20 | ORD-DEN | 11:42 a.m. | 50 | 35 | 0 |  |  |  | X |  | X | Started descent about $1.0 \mathrm{n} . \mathrm{mi}$. late; used higher airspeed to reacquire vertical profile. |
| 21 | DEN-ORD | 3:57 p.m. | -6 | 0 | -500 | X |  |  |  |  | X | ATC requested speed reduction near end of descent. |
| 22 | ORD-DEN | 8:17 p.m. | -6 | 0 | 0 |  |  |  | X | X |  |  |
| Mean |  | . $\cdot$ | 8.39 | 11.39 | 228 | Total |  |  |  |  |  |  |
| Standard deviation |  | . . . . | 35.27 | 15.89 | 702 | 15 | 6 | 4 | 3 | 7 | 11 |  |

## Appendix A-Program Flow Chart























## Appendix B-Calculator Program

Storage Register Location of Descent Algorithm Variables (Numerical Values for the DC-10-10 Model)

| Input Variables |  |
| :---: | :---: |
| 17 | $M_{c}$ |
| 18 | $H_{c}$ |
| 08 | $M_{d}$ |
| 10 | $\mathrm{IAS}_{d}$ |
| 21 | $t_{\text {MF }}$ |
| 22 | $t_{\text {EF }}$ |
| 23 | GW |
| 19 | OAT |
| 07 | $H_{\text {MF }}$ |
| 26 | $\mathrm{IAS}_{\text {MF }}$ |
| 27 | MF ${ }_{\text {DME }}$ |
| 28 | EF ${ }_{\text {DME }}$ |
| 29 | CRS |
| 24 | VAR |
| Computed Variables |  |
| 00 01 |  |
| 02 | Temporary |
| 03 |  |
| 04 |  |
| 05 | $\sum_{j=1}^{i} \Delta t_{j}$ |
| 06 | $\sum_{j=1}^{i} \Delta l_{j}$ |
| 09 | $T_{o}^{\prime} / T_{o}$ |
| 11 | $H_{\text {XO }}$ |
| 12 | $H_{\text {bod }}$ |
| 13 | $K_{\text {gw }}$ |
| 14 | $t_{E, \text { initial }}$ |
| 15 | $t_{E}$ |
| 16 | $\Delta t_{\text {initial }}$ |
| 20 | $T_{\text {trop }}$ |
| 25 | $\Delta t_{\text {req }}$ |
| 55 | $\Delta l_{1}$ |
| 56 | $\Delta l_{2}$ |
| 57 | $\Delta l_{3}$ |
| 58 | $\Delta l_{4}$ |
| 59 | $\Delta l_{5}$ |
| 60 | $\Delta t_{5}$ |
| 61 | $\mathrm{GS}_{5}$ |
| 62 | $K_{\dot{h, ~} \mathrm{XO}}$ |
| 63 | $\mathrm{IAS}_{d, i}$ |
| 64 | GSc |
| 65 | $\dot{h}_{g}$ |
| 66 | $\Delta t_{7}$ |


| Wind | Model Coefficients |
| :---: | :--- |
| 50 | $d D_{w} / d H$ |
| 51 | $D_{w, s}$ |
| 52 | $d S_{w} / d H$ |
| 53 | $S_{w, s}$ |
| 54 | $W_{H, h_{c}}$ |

Airplane Model Coefficients

| 30 | IAS $_{d, \text { min }}$ | $(220$ knots $)$ |
| :--- | :--- | :--- |
| 31 | IAS $_{d, \text { max }}$ | $(350$ knots $)$ |

$32 A_{M}(25750 \mathrm{ft})$
$33 \quad B_{M} \quad(22167 \mathrm{ft})$
$34 \quad C_{M} \quad\left(-1.85 \sec ^{2} / \mathrm{ft}\right)$
$35 \quad A_{I} \quad(-3.07783 \mathrm{ft} / \mathrm{sec})$
$36 \quad B_{I} \quad\left(8.158681 \times 10^{-3} \mathrm{knots}^{-1}\right)$
$37 \quad C_{I} \quad\left(-3.5 \times 10^{-4} \mathrm{sec}^{-1}\right)$
$38 \quad \ddot{x} \quad(-1.3 \mathrm{knots} / \mathrm{sec})$
$39 \quad A_{\mathrm{gw}} \quad\left(-3.863133 \times 10^{-6} \mathrm{lb}^{-1}\right)$
$40 \quad B_{\mathrm{gw}} \quad(2.174392369)$
$41 \quad K_{g} \quad(-1280(\mathrm{ft} / \mathrm{sec}) /$ knots $/ \mathrm{ft})$
$42 \quad K_{\dot{h}, 250} \quad(-24.70908254)$

Computational Constants
4310000
$44 \quad 250$
$45 \quad 3600$
$46-1.978 \times 10^{-3}$
$47 \quad 34293550$
$48 \quad 8900460000$
$49 \quad 177675$

Program Listing

| 81*LBL "PD DCion | 54 PROHPT | 107 FIX 0 |
| :---: | :---: | :---: |
| $02+$ LBL B | 55 ST0 10 | 198 RCL 97 |
| 93 RCL 17 | 56 SF 80 | 109 HHF |
| 64 FIX 3 | 57 "0k" | 110 ARCL X |
| 05 "he" | 58 PROMPT | 111 PROMFT |
| 06 ARCL 8 | 59*LBL D | 112 ST0 9? |
| 97 PROMPT | 69 FIX 4 | 113*LBL F |
| 88 STO 17 | 61 RCL 21 | 114 RCL 26 |
| 99*LBL E | 62 "MF TH ${ }^{\text {a }}$ | 115 -IAS MF |
| 10 RCL 18 | 63 ARCL 8 | 116 ARCL 8 |
| 11 FIX 0 | 64 PROMPT | 117 PROMPT |
| 12 "HE" | 65 ST0 21 | 116 STO 26 |
| 13 RRCL 8 | 66 CF 明 | 119+LBL F |
| 14 PRDMPT | 67 KEC 00 | 120 FIX 1 |
| 1557018 | 684LEL D | 121 RCL 27 |
| 16 "01\%" | 69.RCL 22 | 122 "MF IME * |
| 17 PROMPT | 70 EEF TM" | 123 PRCL $X$ |
| 18+LBL b | 71 ARCL 8 | 124 Prompt |
| 19 FIX | 72 PROMPT | $125 \$ 5027$ |
| 20 RCL 19 | 7351022 | 126 *0K" |
| 21273.15 | 74 XEQ 00 | 127 PROMFT |
| $22+$ | 75*LBL II | 1284LBLG |
| 23 gert | $76 \mathrm{FI} \% 1$ | 129 FIX 1 |
| 24 RCL 17 | 77 RCL 25 | 138 RCL 28 |
| 25 * | 78 "DEL TM | 131 -EF DHE * |
| 2638.96 | 79 ARCL $X$ | 132 ARCL 8 |
| 27 * | 88 PROMPT | 133 PROMPT |
| 28 RCL 18 | $81+$ LEL 00 | 134 STO 28 |
| 29 YEQ 05 | 82 RCL 21 | $135+$ LBL 6 |
| $30+$ | 83 RCL 22 | 136 Fİ 0 |
| 3157064 | 84 Hms- | 137 RCL 29 |
| 32 66c ${ }^{\text {\% }}$ | 85 HR | 138 *MAGCRS * |
| 33 ARCL X | 86 RCL 45 | 139 ARCL 8 |
| 34 PROHPT | 87 * | 140 PROMPT |
| 35 RCL 64 | 8857025 | 141 STO 29 |
| $36-$ | 89 RTN | 142*LBL G |
| 37 CHS | $98+L B L E$ | 143 RCL 24 |
| 38 ST+ 54 | 91. FIX ${ }^{\text {a }}$ | 144 "MAGYAR* |
| 39 "0K" | 92 RCL 23 | 145 RRCL X |
| 46 PROMPT | 93 "614" | 146 PROMPT |
| 41*LBL C | 94 RRCL 8 | 147 STO 24 |
| 42 RCL 08 | 95. PROMPT | 148 "0k" |
| 43 FIX 3 | 9655023 | 149 PROMFT |
| $44 \times \mathrm{Md}$ * | 974 LBL E | 159+LBL A |
| $45 \mathrm{ARCL} \times$ | $98 \mathrm{FI} \mathrm{K}^{\text {1 }}$ | 151 FIM 2 |
| 46 PROMPT | 99 RCL 19 | 1529 |
| 47 STO 98 | 100 "ORT C ${ }^{\text {c }}$ | 153 STO 15 |
| 48 SF | 101 ARCL $X$ | 15436152 |
| 494LEL C | 182 PROMPT | 155 RCL 18 |
| 56 RCL 10 | 103 ST0 19 | 156 - |
| 51 FIX 0 | 184 "0K" | 157 K 6 ? |
| 52 "IASd" | 105 PROMFT | 158 |
| 53 ARCL $\%$ | 1064LBL F | 159 RCL 46 |

168*
161 RCL 19
162273.15
$163+$
$164+$
165 STO 29
166216.65

167 -
168288.15
$169+$
176 LAST:
171 /
172 ST0 99
173 RCL 23
174 RCL 39
175 *
176 RCL 40
$177+$
178 \$T0 13
179 RCL 18
180 XEQ 05
181 STO 01
182 RCL 07
183 XER 05
184 RCL 01
185 -
186 RCL 18
187 RCL 07
188 -
189 /
199 RCL 41
191*
192 ST0 65
193 FS? 00
194 GT0 94
1954LBL 23
196 RCL 17
197 STO 98
198 RCL 26
199286
200 Y) Y?
201 GTO 25
$202 \mathrm{X} \backslash Y$
$203+$ LEL 25
20457063
205 STO 10
206 KEE V4
207 LASTX
208 STO 14
209 RCL 05
210 STO 16
211 XOY
212 K 0 ?

213 GT0 15
214 RCL 31
$215 \$ 7010$
216 XEQ 64
217 LASTX
218 K ?
219 GTO 12
220 GT0 I
221*LBL 15
222 RCL 30
223 RCL 26
224 KY ?
225 XBY
226 STO 10
227 XEQ 64
228 LASTX
229 X 0 ?
230 GTO I
231 LEL 12
232 RCL 14
233 RCL 16
234 RCL 85
235 -
236 •
237 ST0 00
238 RCL 10
239 RCL 63
248-
241*
242 EHTER $\uparrow$
243 ENTER $\uparrow$
244 RCL 14
245 /
2465
247 *
248 RCL 90
249186
258 *
251 SIN
252*
253 -
254 RCL 63
$255+$
256 ST0 10
257 XEQ 04
258 GTO 12
$259+$ LBL 04
260
261 ST0 95
262 ST0 96
263 RCL 10
264 RCL 26
$265 \mathrm{Y} \% \mathrm{Y}$ ?

26657010
267 RCL 10
268 RCL 08
269 /
270 RCL 47
271 *
272 RCL 48
$273+$
274 SQRT
275 CHS
276 RCL 49
277 +
278 RCL 09
279 /
286 STO 11
281 RCL 44
282 RCL 10
$283 \mathrm{X}=\mathrm{Y}$ ?
284 GTO 67
285 RCL 87
286 RCL 43
$287 \mathrm{X}=\mathrm{Y}=$ ?
288 GT0 07
289 ST0 12
290 RCL 07
291 RCL 09
292 *
293 ST0 86
294 RCL 26
295 STO 01
296 RCL 44
297 STO 02
298 XEQ 27
299 RCL 01
300 STO 05
301 RCL 02
302 STO 66
303 STO 55
304 RCL 87
305 RCL 69
366 *
307 RCL 37
308 *
309 RCL 42
310 RCL 65
$311+$
312 RCL 13
313*
$314+$
315 RCL 43
316 RCL 99
317 *
318 RCL 37

319 ＊
320 RCL 42
321 RCL 65
$322+$
323 RCL 13
324 ＊
$325+$
326 ／
327 LN
328 RCL 37
329 ／
338570 日1
331 ST＋ 05
332 RCL 97
333 RCL 43
$334+$
3352
336 ；
337 RCL 99
338 ＊
339 ST0 92
340 RCL 44
341 XVY
342 XEQ 0.3
347 RCL 82
344 KE0 95
345 ＋
346 RCL 01
347 ＊
348 RCL 45
349 ／
350 ST＋ 06
35157056
352 RCL 44
353 ST0 01
354 GT0 08
3554LBL 07
356 RCL 97
35757012
358 日
35957055
360 §T0 56
361 RCL 26
362 ST0 01
363 ＋LBL 98
364 CF 95
365 RCL 11
366 RCL 18
367 X $>$ Y？
368 CTO 99
369 STO 11
370 SF 85
371 LBL 99

372 RCL 11
373 RCL 12
$374 X>Y$ ？
375 GTO 10
376 RCL 89
377 ＊
378 STO 00
379 RCL 10
386 STO 02
381 XEQ 27
382 RCL 01
$383 \mathrm{ST}+65$
384 RCL 62
385 ST0 57
386 ST＋ 86
387 RCL 10
388 RCL 36
389 ＊
390 ETY
391 RCL 35
392 ＊
393 RCL 65
$394+$
395 RCL 13
396 ＊
$3975 T 001$
398 RCL 12
399 RCL 09
409＊
401 RCL 37
482 ＊
$403+$
404 RCL 11
405 RCL 89
406 ＊
497 RCL 37
408 ＊
499 RCL 01
$410+$
411 ／
412 LN
413 RCL 37
414 ／
$415 \mathrm{ST}+95$
416 STO 91
417 RCL 11
418 RCL 12
$419+$
4202
421 ／
422 RCL 09
423 ＊
424 STO 82

425 RCL 10
426 XCY
427 XEO 03
428 RCL 02
429 XEQ 05
$430+$
431 RCL 01
432 ＊
433 RCL 45
434 ／
$435 \mathrm{ST}+06$
436 ST0 58
437 GTO 11
438 LELL 10
439 日
440 STO 57
441 ST0 58.
442 RCL 97
443 STO 11
444＋LBL 11
445 FS？ 05
446 GTO 21
447 RCL 88
448 RCL 32
449 ＊
458 RCL 33
$451+$
452 STO 01
453 RCL 18
454 RCL 99
455＊
456 －
457 CHS
458 RCL 34
459 ／
468 SQRT
461 STO 02
462 RCL 11
463 RCL 99
464 ＊
465 RCL 81
466 －
467 RCL 34
468 ／
469 SQRT
470 RCL 65
471 －
472 ST0 62
473 RCL 82
474 RCL 65
475 －
476 ／
477 LN

478 RCL 65
479 *
480 CHS
481 RCL 62
482 -
483 RCL 65
484 -
485 RCL 02
$486+$
4872
488 *
489 RCL 34
498 *
491 RCL 13
492 /
$493 \mathrm{ST}+05$
494 ST0 60
495 STO 01
496 RCL 11
497 RCL 18
$498+$
4992
500
501 RCL 89
502 *
503 ST0 04
584 RCL 88
565 K
506 XEe 06
507 RCL 84
508 KEQ 05
$509+$
510 STO 61
511 RCL 01
512 *
513 RCL 45
514 /
$515 \mathrm{ST}+06$
51657059
517 RCL 96
518 STO 01
519 GTO 22
$520+$ LBL 21
5218
522 ST0 59
523 RCL 10
52457001
$525+$ LBL 22
526 RCL 17
527 STO 02
528 RCL 18 529 RCL 89 530 *

53157000
532 XED 27
533 RCL 61
534 ST 95
535 RCL 02
536 ST 06
537 RCL 28
538 RCL 27
539 -
540 RCL 06
541 -
$542 \times$ < $?$
543 XED 02
544 RCL 45
545 *
546 RCL 03
547 RCL 64
$548+$
549 ;
559 STO 66
$551 \mathrm{ST}+05$
552 FS? 0 的
553 GTO 18
5545
555 RCL 95
556 RCL 25
557 -
558 ST0 15
559 ABS
568 XPY ?
561 RTN
562 GTO J
5634 LBL 18
564 RCL 65
565 ST0 25
566 RCL 45
567 /
568 HMS
569 RCL 22
578 HMS+
571 STO 21
572 GTO J
$573+$ LBL I
574 FIX 4
575 RCL 15
576 RCL 45
577 /
578 MMS
$579 \times 10$ ?
580 GT0 26
581 "LATE -
582 ARCL Y
583 PROMPT

584+LBL 26
585 CHS
586 "HOLI"
587 ARCL X
588 PROMPT
$589 * L$ LL J
590 RCL 27
591 RCL B6
$592+$
593 FIX I
594 "IDL DME -
595 RRCL
596 PROMPT
597*LBL J
598 RCL 66
599 RCL 45
608 /
601 HM
602 RCL 22
603 HMs
604 FIX 4
685 "IDL TM *
666 ARCL $X$
607 PROMPT
688+LBL 27
609 RCL 01
610 XEQ 13
61157001
612 RCL 82
613 XEA 13
614 STO 02
615 ST0 03
616 ST- 01
617 RCL 01
6182
619 /
620 ST+ 92
621 RCL 01
622 RCL 38
623 /
624 ㅈ0?
625 CHS
626 STO 01
627 RCL 90
628 KEO 85
62957094
630 RCL 02
$631+$
632 *
633 RCL 45
634 /
63557002
636 RTN

637*LBL 85
638 STO 04
639 RCL 52
640 *
641 RCL 53
$642+$
643 RCL 04
644 RCL 50
645 *
646 RCL 51
$647+$
648 RCL 29
649 -
650 RCL 24
$651+$
652 cos
653 *
654 CHS
655 RCL 94
656 RCL 54
657 *
658 RCL 18
659 /
660 -
661 RTH
$662+$ LBL 29
663 RCL 0 日
664*LBL 93
665-12 E-06
666 *
6671
$668+$
669 /
670 RTN
$671+$ LBL 13
6721
673 KYY
674 X ${ }^{2} Y$ ?
675 GTO 29
676 RCL 09
677 +LBL 66
67836152
679 -
680 RCL 46
681 *
682 RCL 20
$687+$
684 RCL 20
$685 \mathrm{KK}=\mathrm{Y}$ ?
686 X KY
687 SQRT
688 RCL 2
689 *
69938.96

691 *
692 RTH
693*LBL 12
694 CHS
695 FC? 80
696 RTH
697 FIX I
698 RCL 28
$699+$
700 ST0 80
701 RCL 65
702 RCL 45
703 /
704 HMS
785 RCL 22
706 Hist
707 STO 21
708 RCL 00
709 "EF〈"
710 ARCL $X$
711 PROMFT
712 LLBL 39
713 PROMFT
7144LEL H
715 \$T0 96
716 RCL 27
717 KYY
718 XY ?
719 GTO 46
720 RCL 27
721 ST0 12
72255
723 STO 63
7241
725 ST0 04
7264LBL 30
727 RCL IND 03
728 ST+ 02
729 RCL 82
736 RCL 66
$731 \times 1=Y$ ?
732 GTO 32
7331
$7345 T+64$
7355
736 RCL 84
$737 \mathrm{X}\rangle \mathrm{Y}$ ?
738 GT0 31
7391
740 ST+ 63
741 GTO 30
742 LBL 46

743 RCL 87
744 RCL 69
745 *
746 STO 03
747 RCL 26
$748 \mathrm{~K}\rangle$
749 XEQ 83
750 RCL 03
751 XEQ 85
$752+$
753 RCL 27
754 RCL 86
755 -
756 /
$7571 / 2$
758 RCL 45
759 *
760 RCL 37
761 *
762 RCL 13
763 *
764 ETX
765 STO 01
766 RCL 26
767 RCL 36
768 *
769 Et
770 RCL 35
771 *
772 STO Y
773 RCL 37
774 RCL 83
775 *
776 +
777 RCL 61
778 *
779 -
780 RCL 37
$781 /$
782 RCL 89
783 /
784 CHS
785 GTO 14
7864 LBL 31
787 RCL 16
788 GTO 14
789 LBL 32
790 RCL 88
791 RCL 82
792 -
793 RCL IND 03
$794+$
795 ST0 01

79632
$797 \mathrm{ST}+84$
798 GTO IHD 64
7994LBL 33
800 RCL 07
801 GTO 14
802 4 LEL 34
863 RCL 43
884 RCL 67
805 -
886 RCL 01
887 *
808 RCL IND 03
809 \%
816 RCL 97
$811+$
812 GT0 14
813*LBL 35
814 RCL 12
815 GT0 14
8164LBL 36
817 RCL 11
818 RCL 12
819 -
820 RCL 01
821 *
822 RCL IND $\operatorname{GT}$
823 ;
824 RCL 12
$825+$
826 GTO 14
827*LBL 37
828 RCL 62
829 RCL 13
830 *
831 RCL 09
832 /
833 RCL 60
834 *
835 RCL 11
$836+$
837 RCL 16
838 -
839 RCL 60
$848 \mathrm{x} \uparrow 2$
841 /
842 CHS
843 STO 09
844 RCL 45
845 RCL 01
846 *
847 RCL 61 848 /

849 ST0 61
850 X 12
851 RCL 00
852 *
853 RCL 62
854 RCL 13
855 *
856 RCL 89
857 ;
858 RCL 01
859 *
$869+$
861 RCL 11
$862+$
8634LBL 14
864 FIM 日
865 "H = "
866 ARCL X
867 GTO 39
868+LBL a
369 EREE 01
870 CLE
871 EREG 11
872 CLE
873 CF 87
874 CF 66
$875+$ LEL 49
876 *H=? FT"
877 PROMPT
$878 \times 0$ ?
879 GT0 41
880 STO 0
881 "DIR . SPD ?"
882 PROMPT
883 STO 50
884 FRC
8851900
886 *
887 RCL 00
888 KPY
$889 \mathrm{E}+$
899 RCL 11
891 ST0 01
892 RCL 12
893 ST0 02
894 RCL 15
895 ST0 03
896 RCL 00
897 LASTK
898 ع-
899 RCL 04
900 STO 11
901 RCL 05

982 STO 12
983 RCL 66
9045 TO 15
905 RCL 50
986 IHT
907 YEQ 42
908 RCL $B 8$
$909 \mathrm{X} \backslash Y$
$918 \Sigma$
911 RCL 11
912 STO 04
913 RCL 12
91457095
915 RCL 15
916 STO 96
917 RCL 01
918 STO 11
919 RCL 92
92857012
921 RCL 93
$9225 T 015$
923 GTO 40
$924+$ LBL 41
925 RCL 13
926 RCL 01
927 *
928 RCL 16
$929 X=8$ ?
930 XER a
931 /
932 RCL 03
933 -
934 RCL 13
$935 \mathrm{Xt}+2$
936 RCL 16
937 /
938 RCL 14
939 -
940 STO 11
941 /
942 STO 52
943 RCL 13
944 RCL 16
945 ;
946 STO 12
947 *
948 CHS
949 RCL 01
950 RCL 16
951 /
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954 KCL 13

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955 RCL 04
956 *
957 RCL 16
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959 RCL }6
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961 RCL 11
962;
963 ST0 50
964 RCL 12
965*
966 CHS
967 RCL }0
968 RCL 16
969 %
970+
9715T051
972 6
973 STO 54
974 "HIND IN*
975 PROMPT
9764LBL 42
977 300
978 X\>Y
979 X>Y?
980 GTO 43
98160
982 X()Y
983 X>Y?
9 8 4 ~ R T N ~
985 FS? }9
986 RTN
987 FS? }6
988 GT0 44
989 SF }8
990 RTH
991*LBL 43
992 FS? }0
993 RTN
994 FS? 06
995 GTO 45
996 SF 97
997 RTN
998*LBL 44
999360
1000 +
1001 RTH
1002*LBL 45
1003 360
1094 -
1085 RTH
1096 .END.
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Figure 1. Vertical-plane geometry of computed descent path.


Figure 2. Logic flow of flight-management descent algorithm.


Figure 3. Schedule selection of indicated airspeed descent via interpolation.


Figure 4. Quadratic curve fit of altitude plotted against time for DC-10 airplane executing a constant 280 -knot indicated airspeed descent.


Figure 5. Vertical-speed model of DC-10 airplane for constant indicated airspeed descents (idle thrust).


Figure 6. Modeled vertical speed at sea level (idle thrust) for DC-10 airplane.


Figure 7. Flight data and generic parabolic model with $c_{0}=-1.85 \sec ^{2} / \mathrm{ft}$ for constant Mach number descents (idle thrust) for DC-10 airplane.


Figure 8. Model of $c_{1}$ for constant Mach number descents for DC-10 airplane.


Figure 9. Modeled and actual true airspeed reduction in level flight (idle thrust) for DC-10 airplane.


Figure 10. Vertical speed at sea level as a function of gross weight for DC-10 airplane.


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Figure 11. Programmable descent calculator.

(a) Metering-fix crossing errors due to temperature error.

Figure 12. Metering-fix crossing errors.

(b) Metering-fix crossing errors due to gross-weight variation.

Figure 12. Continued.





(c) Metering-fix crossing errors due to descent Mach number error.

Figure 12. Continued.

(d) Metering-fix crossing errors due to descent airspeed variation.

Figure 12. Continued.

(e) Metering-fix crossing errors due to wind-gradient variation.

Figure 12. Concluded.


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